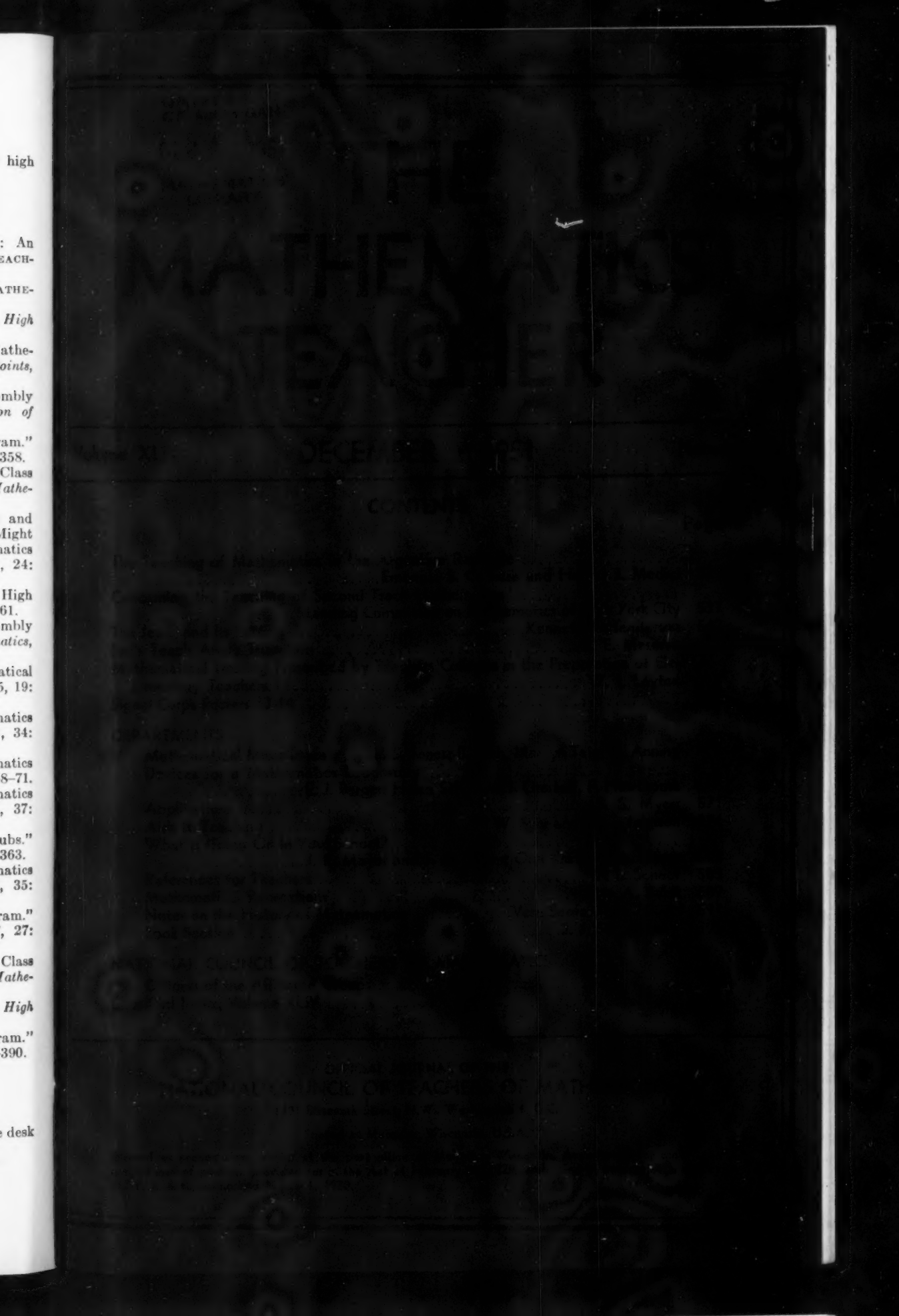

THE MATHEMATICS TEACHER

Volume XLIV Number 8

The Teaching of Mathematics in the Argentine Republic

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THIS ARTICLE is addressed to the North American colleagues who feel curious to know what, how, and to whom mathematics is taught in the Argentine Republic.

Last year we visited their hospitable country with the same curiosity, attending more than fifty classes in high schools, colleges and universities, to see with our own eyes how this subject was taught after a war that had clearly shown the necessity for every citizen to know more mathematics. With this experience, and a description of the aims and characteristics of our mathematics curriculum, we wish to show some differences existing in the teaching of that subject in these two American countries.

WHAT DO WE TEACH IN OUR ELEMENTARY SCHOOLS?

Although education is divided into elementary, secondary, and university levels, the same as in the United States, the elementary teaching is the only one obligatory in Argentina for girls and boys 6 to 12 years of age. It has a duration of seven years and is directed towards giving children that basic knowledge which is indispensable for practical life. We shall limit our writing to the mathematics program.

In the elementary schools, mathematics is taught daily during the seven years. The following subjects are developed:

(a) Oral and written numeration, and the four fundamental operations with whole numbers, decimal and common fractions.
(b) The metric system of weights and measures.
(c) Solving problems with the contents of (a) and (b).
(d) Simple and compound rule of three.
(e) Problems of interest, discount and proportion.
(f) A knowledge of plane and solid figures; drawing of perpendicular and parallel lines with ruler, square and compasses. Construction of triangles, parallelograms, regular polygons, etc.
(g) Calculation of the area of a triangle, parallelogram, trapezoid, regular polygons, etc.
(h) Length of the circumference, and area of the circle.

(i) Area and volume of such solids as the cube, right parallelepiped, prism, cylinder, pyramid, cone and sphere.

The teaching of these topics is presented intuitively, avoiding formal definitions and proofs, but having the pupils solve the problems in a reasonable way. The totality of this mathematics knowledge is similar to what we believe is known in the United States as an elementary social or general mathematics.

Some Characteristics of Our Teaching

Teaching in our country has tended, little by little, to become uniform for all schools. The national government permits, in an absolutely free character, the establishment of secondary and university teaching throughout the country. The provinces (which are the divisions corresponding to the states of the Union) finance a program of free elementary education in their own territories. Beyond this, the national government supports the schools of the federal capital (Buenos Aires City), the national territories, as well as a great number of Schools of Campaign established in the provinces.

As a result of this national support, the elementary teaching plans are practically the same in the whole Republic. Provincial and private programs agree with those of national schools, principally because it is required that the last course of the elementary school must agree with the national official program in order that pupils be able to enter the secondary schools.

In the secondary schools the uniformity is now absolute, because the private secondary schools must adjust themselves to the official program in which teachers check their pupils by national examinations. This rigid system also differs from the North American system in that all students in each program of studies have to pursue the same subjects to get their diplomas. The system of credits does not exist in Argentina. The application of this uniform criterion for graduation is supported by the following arguments:

(a) It is desired that all the students have the same minimum of preparation.

(b) The student generally is not capable of making an intelligent choice of subjects, the content of which he does not know.

(c) The principal aim of the secondary teaching is to educate, and a student cannot escape prescribed study merely for reasons of dislike or antipathy developed in initial study, or for simple prejudices. (We believe that the second world war proved, in the United States, that the soldiers and industrial workers must know more mathematics than that which had been taught to them.)

Argentina's culture, small or large, is of European roots (especially French and Italian), and in these countries mathematics has always been one of the classical foundations of all levels of education. Our secondary schools aspire to give a general knowledge that allows the young students to continue university study, if they wish to, but not without some knowledge of the elementary notions of the other subjects that contribute to general culture.

We have had the satisfaction of noting that in all the universities we visited, including Northwestern, Harvard, Columbia, Chicago and even the Massachusetts Institute of Technology, which is predominantly technological, the teaching of mathematics had a cultural nature, and many courses of abstract mathematics were offered. The compulsory courses in "foundations of mathematics" for all students of the University of Chicago is another comforting example that mathematics has a classic place in the education of youth.

What Do We Teach in the Secondary Schools?

There are four fundamental types of secondary schools. They prepare:

(a) Graduates with high school diploma ("bachilleres").
(b) Teachers for elementary schools (graduated in normal schools).
(c) Commercial experts (graduated in commercial schools).
(d) Industrial technologists.

The "bachilleres" attend schools called "Colegios Nacionales" (literary national colleges) corresponding to an American high school for boys, and "Liceos de Señoritas" (Lyceum for girls). These schools have two divisions, basic and superior (something like junior and senior) of three and two years respectively. In our country, with some few exceptions, pupils of each secondary school are of the same sex.

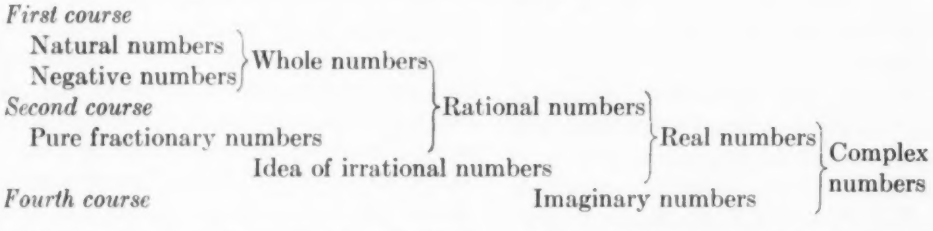
In all five years of secondary school, mathematics study is compulsory for all students. There must be at least four periods of 45 minutes each week during the whole scholastic year which is from thirty to thirty-two weeks, each week having six school days. For national colleges and lyceums, the study is distributed in the following way:

First year: Arithmetic, first part; plane geometry, first part.
Second year: Arithmetic, second part; plane geometry, second part.
Third year: Algebra, first part; plane geometry, third part.
Fourth year: Algebra, second part; solid geometry.
Fifth year: Plane and spherical trigonometry; cosmography.

Arithmetic and geometry are always taught by the same teacher in each course in alternative classes.

Since 1926 the mathematics programs follow the logical development shown below, assuming those mathematical properties for which a logical proof is too difficult for the students to understand. Program instructions say: "The teacher can omit a demonstration rather than give a deficient proof."

Here is the logical order of the arithmetic program.



Some details concerning the first course arithmetic program for the national colleges and lyceums are:

(a) It is assumed that the students know the set of natural numbers, and have an idea of the decimal system of notation. (From the beginning, the literal representation of numbers is used to teach by symbols the operations with numbers and their properties.) Graphical representation and the Roman system of notation are also taught.
(b) Addition of natural numbers; the symbolical representation, and examples of its properties.
(c) Subtraction of natural numbers; the symbolical representation and its properties. Proof of the law of monotony is given in order to initiate the students to proofs by "reductio ad absurdum." (Indirect method of proof.)
(d) Algebraic sums (combined additions and subtraction of natural

numbers). Use and removal of parentheses. Transposition of terms in an equation.

(e) Multiplication of natural numbers: definitions and properties. Here for the first time examples of the direct method of proof are given in establishing the distributive law.

(f) Division of natural numbers: definitions and properties. Some proofs applied to the study of subtraction.

(g) Transposition of factors and divisors in an equality.

(h) Powers of natural numbers: definitions and properties. (The students' only previous knowledge of this subject has been with square and cube.)

(i) Square root: definitions, examples and rules to extract the square root of a number greater than 100.

(j) Prime and compound numbers. H.C.F. and L.C.M. Rules to find them mentally or by using factors.

(k) Negative numbers: the necessity of their creation in order to make the subtraction of natural numbers always possible. Practical examples (profit, loss, etc.). One whole number is equal to, less than, or greater than another.

(l) Addition, subtraction, multiplication, division, powers and roots of whole numbers.

In Argentina the teaching of geometry is inspired by the works and experiences of the Italian geometers of the first two decades of this century, and especially those of the great geometer Federigo Enriques. We believe that our programs of geometry, which have been used with successive revisions since 1926, are a real success from the points of view of content and of mathematical rigor, both considered with respect to the age of the students, and teaching principles. In this program students quickly form their geometric vocabulary and learn to employ a simple and exact language. By simple experiences they discover the properties of the figures. Then the students are guided by the teacher to the discovery of the rigorous proof of these properties.

We have always found that students learn geometry easily, because it agrees with their spatial intuition (even in solid geometry), while in the United States there is reluctance to teach geometry, because, as we heard it said very often, it is beyond the level of the mentality of a great percentage of pupils. It is very simple, as everybody knows, to make the students discover properties of plane figures by measuring, folding paper models, superposing cardboard figures, etc., and properties of solids by observing relations between walls and edges in the classroom. This opinion on learning geometry is supported by our experience of twenty-eight years of teaching, and it depends, perhaps, on the way we teach, that is, recurring to all previous experiences, permitting the student to discover properties by himself, and then guiding him, with opportune questions, to the proof of these properties.

We never permit a student to study or learn a lesson for the first time by the use of the textbook. First the teacher teaches the lesson to him in the classroom. At the next class meeting there is collaborate activity on the lesson by both the teacher and the students. It is in this second lesson that the pupil must review, preparing to show his learning on the blackboard. During this review the pupil may use his textbook.

How Do We Teach?

The method of teaching mathematics in the secondary schools is optional with each teacher. However, as a rule it is demanded that the good teacher must teach in each class with the active participation of all students, the lesson for the next class. It is assumed that the majority of students have not reached a state of learning where they can assimilate, by themselves, the lesson in the textbook. Even for solving exercises and problems assigned for homework,

the teacher must teach, during the class, with the active participation of all the students, one or two examples which will serve as a guide for doing his homework. For young pupils who must learn five different lessons every day, there is a real need for this method of teaching.

In our visit to the United States, three quite common practices attracted our attention. The classes always began by giving the assignment for the next class; the students were permitted to have their textbooks open during the class; there was an excessive use of tests. We stress that, in Argentina, the lesson for the next class is that one learned in the former class.

It is not obligatory for a student to have a textbook. If he has one he is forbidden to keep it open on his bench, because his duty is to go to school knowing his lesson and with the homework done in his copybook. A teacher permitting open books in his classes would be criticized even by his own students. According to our customs the student can only bring exercises written in a paper or a special book of exercises.

Periodically we give written examinations and in these cases it is prescribed that the questions must be on past lessons that are quoted in the book of topics that each professor has written during his former classes. In these examinations the teacher can also propose exercises resembling those solved in class, or a theorem that the pupil must demonstrate with all the details of a perfect mathematical proof.

What are the methods used? That depends, generally, on the schools from where the teachers have graduated.

According to official regulations for certifying teachers in the secondary schools, mathematics can be taught by graduates from the mathematics department of the National Institute of the Secondary Professorship, graduates in science in a normal school for teachers, and graduates in faculties of sciences.

We shall report only on the methods prescribed by the National Institute of Secondary Professorship, because we are among its graduates and we think it has given best results. The first Institute was created in Buenos Aires City in 1904, and its direction and teaching was confined to professors who had specialized in each specific area through study in Germany. The Institute of the City of Buenos Aires, actually consists of 12 departments, one of which is the mathematics department. It has a professor of mathematical methodology and teaching practice, whose duty it is to develop "the art of teaching." The method recommended, which we have always practiced during our teaching career, and which we recommend to our students in the normal school for teachers "Mariano Acosta," is the "heuristic" or "genetic" or the rediscovering by the pupils.

A classic work of methodology in Argentina is "The Teaching of Mathematics in the Elementary and Secondary Schools" by Professor J. W. A. Young, former Emeritus Professor of the University of Chicago. We agree with this distinguished teacher, who visited Europe in 1900 to study the teaching in the secondary schools there, that the heuristic method is the best, not to say the unique one that should be employed in teaching mathematics in the secondary schools.

In fact, if the statements of mathematics propositions are never imposed, nor previously studied in a textbook, the students' experience in observing appropriate examples and abstracting through answers to a chain of questions awakens confidence in themselves and a tendency to correct thought.

This method gives the teacher a daily check of the interest and intelligence of each of his students, and, what is more important, tests his efficiency as a teacher. This method also enables the teacher to fill in the blanks in the preparation of each pupil, putting the whole class at the same level in a short time.

We had the pleasure of observing very good classes taught in this way in two high

schools of science for bright pupils in New York City. We appreciated the activity and gaiety of these classes, in contrast to the silent classes where tests were given or exercises were solved or discussed.

The National Institute of the Secondary Professorship was initiated by foreign teachers, and the heuristic method was established. In 1926 the official program was revised in accordance with the inspiration of the teaching of mathematics in Europe, and gives the subjects to be taught in the secondary schools of the Argentine Republic. This revision met with great success, thanks to the capacity and devoted service of a distinguished Argentinian teacher, Professor Florencio D. Jaime, teacher of mathematics methodology and projective and descriptive geometry in the National Institute of the Secondary Professorship, author of the mathematics program of 1926, and retired as General Inspector of Secondary Teaching a few years ago.

Under his direction, the heuristic method, which until this time had been taught in the Institute in a formal manner, was improved because Professor Jaime insisted that the teacher must not employ one method exclusively to instruct his students. Following his teaching, we now distribute the 45 minutes of a class in the following way:

(a) 10 minutes to review the former lesson.
(b) 25 or 30 minutes to teach the new theme.
(c) 5 or 10 minutes to a general review of this new theme.

We now employ the heuristic method only to teach the new topic, in which there is developed the statement of definitions, the discovery and demonstration of a theorem, and the solutions of exercises or problems.

Under the heuristic method the student will learn and express his knowledge only with the teacher as a guide. This deficiency is now corrected by the use of the expository method in part (a). That is, in order to get the student to express himself, to put order into his ideas, and to give him the chance to become qualified, we now devote the first 10 minutes in all classes to a review of the theme of the last class, which, as we said before, is the obligatory lesson.

As a student recites, the other students of the class are judging the qua ity of his recitation, and correcting him, under the teacher's direction, if that student makes mistakes. In such a situation the teacher will by his own teaching demonstrate the art of giving a good lesson, demanding accurate drawings, judiciously employing the blackboard, correcting the mistakes of language and of logic if the other students have not perceived it, and he will ask all the necessary questions to determine that the student really understands the material about which he is speaking.

At other times, the teacher also employs the recitation to recall the several properties learned in former courses, especially at the end of the scholastic year when a general review of the course is made before the final examinations. When a student persists in an incorrect conclusion, supporting his deductions with drawings, (as for instance in a triangle that looks isosceles when there is no reason for it to be isosceles) the socratic method can be used to help the student discover his error.

It is thus evident that the group of teachers graduated from the National Institute of the Secondary Professorship, use different methods than those we observed in many classes in the United States.

Through daily employment of this method of collaboration of pupil and teacher, which Professor Young refers to as "talk and chalk," we have developed a high percentage of good pupils in mathematics; an absolute control upon each of them; a natural discipline; and no loss of time or coverage of the program.

But what is the role played by the textbook? Merely to have clearly printed that which is developed by the teacher on the blackboard, and to give sets of exercises,

in order that the students may prepare their lesson and homework after a first instruction received in class. Some teachers prefer not to use textbooks, making the students take notes. But these notes are generally incomplete and defective, and the teacher must correct them, and during the class the student usually is more preoccupied in taking notes than in following the theme of the lesson.

Whom Do We Teach?

We teach all students, girls and boys, of age 12 or more, who have passed the seven grades of the elementary school, and, in addition, the entrance examination to the secondary school. This examination was introduced in 1936 in order to solve, in a democratic and equitable way, the problem that arises when the number of candidates is greater than the vacancies in the schools.

The questions on these examinations refer to the subjects of Spanish (our official language) and mathematics, considered to be learned in the previous training in an elementary school. We do not try to measure their aptitude, because in Argentina it is assumed that any student who has passed the elementary school work is prepared to follow the secondary program.

Until we arrived in the United States we had not thought of the problem of the "slow" students, that is, those who do not have the capacity to learn mathematics, especially solid geometry. We had not thought of it because we do not face this problem in Argentina.

We always find some "bright" students in our classes, and the heuristic method brings forth many "mathematics inventors." For us, "bright" students are those who intuit, infer, and retain easily, who invent methods to solve problems and give demonstrations. Even though sometimes defective or even false, the inventions reveal the students' creative urge and their originality.

We do not say that we have no "bad" students who very often do not know their lessons, but generally it is the students' fault, because they do not study at home, because they do not solve exercises, or plainly because they do not review their lessons, and do not pay attention in class.

Each class has students who are not well trained in former courses or in the elementary school, other students who have forgotten nearly all they were taught, and even some students who have failed in mathematics in former courses. These students interfere with the purpose of carrying on the course in the manner described before, but the heuristic method allows the teacher to pay more attention to these students, answering their questions more frequently than those of the others in the class.

The parents are also requested to watch over the students during the study hours at home, and very often these deficient students make remarkable progress. In Argentina the student spends more time with his parents than in the United States. Because classes are held in a single session they have lunch at home, and they return to school for the other session only twice a week for a couple of hours, for physical culture or handwork.

The secondary teaching is not compulsory. It is directed to the average ability of students. The boys and girls without interest or of a very low level of intelligence quickly fall behind the others and they end by dropping their studies.

The "bright" students usually jump over intervening courses by taking special examinations, written and oral, as "free students." The examinations are given a short time before the beginning and after the end of the scholastic year. These examinations can also serve as make-up tests for students who failed, and for those with incomplete study due to illness or other reasons.

Summary

In short we can say:

1. Many of those who teach mathematics in Argentina are specialized teachers for the teaching in secondary schools.
2. We teach a new subject by the heu-

ristic method, but we also use the recitation, examination, and socratic methods when necessary.

3. In the scholastic curriculum, mathematics occupies the position of a formative subject, serving principally as a model of logical and deductive reasoning, without neglecting the practical uses in daily life.

4. We always teach arithmetic and geometry, algebra and geometry, or trigonometry and cosmography in the same course.

5. The teaching of arithmetic is done in a genetic way: natural numbers, negative numbers, whole numbers, rational, irrational, real and complex numbers using literal representation from the very start.

6. The instruction in geometry follows the Italian school.

7. After the second or third month of the class in the first course, we begin to give simple but rigorous demonstrations in arithmetic and geometry, making the pupils learn the reasons of each step and know how to express themselves properly on the blackboard or in written lessons.

8. Solid geometry, which is taught in the fourth course for the bachelor's degree and the school teachers' degree, and in the third course of commercial school, never gives us trouble.

9. Teaching in secondary schools is not obligatory; following the bachelorship those pupils who desire, may go to the university, others may wish to have only the general culture secured through the instruction.

10. The secondary teaching is uniform in all the territory of the Argentine Republic and it is supported directly by the national government.

11. The student cannot only choose the type of school he wishes to attend, but he can also transfer to others by taking complementary examinations.

12. We do not give special classes for discussion because discussion comes naturally from the teacher's questions and from the students' answers, doubts, or expressed opinions.

13. General mathematics education is implicitly contained in the elementary school, and especially in the arithmetic program of the second course, where the calculations of areas and volumes are applied to solve problems of daily life.

14. With the study of the cosmography, the pupil has the opportunity to apply his knowledge of mathematics, physics, and chemistry learned in former courses.

15. The application of the heuristic method gives the pupil a feeling of elaboration of his own knowledge, and the pleasure of knowing that he really learns at school even without a textbook.

These are the main points of view regarding instruction in mathematics in Argentina, and they are in some cases different from those we observed in our short visit to the United States. The visit of North American teachers to Argentina during their summer vacations, which coincides with the season in which our programs are in full operation, would provide an opportunity to inform opinions about our defects and virtues, and it would contribute to the formation of bonds among the members of the teaching family of both Americas, which until today have long been missing.

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Concerning the Teaching of Second Track Mathematics

A Report Prepared by the Standing Committee on Mathematics of New York City*

I. Introduction

The problem of adapting our courses in mathematics to groups of varied abilities, interests, and needs has been with us for a long time. Our most recent attempt in this area has been the experiment on the Ninth Year Second Track Mathematics.

The report of the Committee on Ninth Year Second Track Mathematics was issued in June 1948. This report reviewed and stated the problem, discussed the question of the selection of the students for the course, analyzed the characteristics of the students, made some general suggestions on methods of teaching, outlined the aims and objectives of the course, and presented a tentative order of topics (with time allotments) to be covered in 28 weeks.

A workshop of teachers from the Junior High Schools, the Senior High Schools, and the Vocational High Schools then worked on this report during the Fall 1948 term. The workshop produced an exceptionally valuable set of "Teacher Notes" to accompany the syllabus contained in the original report. Objectives, procedures, sample exercises and suggested projects for individual units of work, plus an additional section on mathematical recreations, comprise these "Teacher Notes."

The Spring 1949 term was used as a try-out period for the Second Track course. Eight Junior High Schools, ten Senior High Schools and four Vocational High Schools taught the syllabus to approximately 4,000 pupils. Additional instructional materials were prepared and issued to the teachers by Mr. Harry Sitomer, who was assigned to coordinate the work of all the schools. This syllabus will now be introduced generally in all the schools.

The purpose of this report is to underline for all teachers of mathematics some general questions of approach and attitude toward the whole problem, as well as to list a number of specific and concrete suggestions on classroom procedure.

II. Characteristics of the Second Track Pupil

The guiding philosophy for second track mathematics, the formulation of aims and objectives for this course, the selection of subject matter, and the methods of teaching must all be determined by the type of pupil for whom a second track course is necessary.

It will be found that these pupils are frightened and inhibited by the ordinary aspects of mathematics. They have had, in many instances, rather unhappy careers in school. The problem for the teacher is to rebuild the self-respect of these pupils, to imbue them with more favorable attitudes toward school and to convince them of the worthwhileness and practical value of mathematics.

While in many respects the slow learners are much like the average or superior groups, the qualities which really differentiate them have to do with the higher mental processes. The slow pupil, as a rule, is weak in forming associations between words and ideas; he cannot generalize as readily as the average or superior pupil; his memory is relatively poor; he has little aptitude for dealing with abstract

* The members of the committee were: Irving Cohen, Antonia Higginson, Julius H. Hlavaty, Michael M. Katzoff, Daniel Malament, Maurice Nadler, Harry Sitomer, Alan Wayne, Catherine Wilson, and Theresa Molloy, Chairman.

material and learns most readily through a concrete inductive presentation.

Slow pupils have a shorter attention span than normal pupils; they cannot concentrate so long on a single topic. Their interest may be quite as high in the initial presentation of a topic but may wane almost at once. While they need many exposures to a mathematical principle for any kind of mastery, they react unfavorably to routine repetition and ordinary drill. Since they do not make generalizations as readily as the average or superior groups, there is, consequently, less transfer to related or similar fields. These pupils must therefore be given a direct appreciation of the role of mathematics in understanding, controlling, and improving our environment. Finally, they, perhaps more than any of the others, need to acquire a sense of accomplishment and to gain the satisfactions of success.

III. The Elements of a Guiding Philosophy

The problem of the increasing number of failing pupils in mathematics has been with us since the high schools were opened to all the youth. Two major attempts to solve the problem have been made. The first constituted a denial of the existence of the problem with the statement to the pupil: "If you can't succeed in the mathematics we offer, then elect some other subject." The second consists of "watering down" the curriculum with the hope that the removal of syllabus difficulties will enable some of these pupils to receive passing grades. In this attempt, parallel courses in plane geometry have been prepared for students who are not expected to take the Regents' examinations.

The first attempt to solve the problem disregards our obligation as teachers to prepare youth for life in a technical society in which the applications of science and mathematics are assuming larger and more significant proportions, and in which a citizen is expected to make sound civic judgments based on a knowledge of facts—a knowledge which in part is given by science and mathematics. To disregard the need for a familiarity with the methods, concepts, and contributions of mathematics, is to handicap any individual in his role as a citizen and to permit the perpetuation of ignorance in a large section of the population. No democracy can evolve to its destiny as a nation of free and secure individuals, while a large section of the population is doomed to a second class education.

The second attempt also fails. This may be seen in terms of objectives and needs. These needs may be summarized as follows: the revival of a lost interest in mathematics; the removing of a depressing sense of failure; the restoration of self-respect; the acqusition of the fundamental concepts of mathematics which are necessary for the adequate functioning of an individual of our democratic society—and the acquisition of the concomitant arithmetic skills. How does a watered down syllabus meet these needs? The history of our present (Regents) syllabuses indicates that they were made to prepare students for college studies in the sciences and mathematics. Thus, problem-solving techniques are emphasized. Topics which are vital for college mathematics receive extended treatment on a technical level, e.g. equations, graphs, formulas, irrationals. We also find such topics as complex fractions, logarithms, the binomial theorem, and loci—all helpful in preparing the student for the calculus. In geometry, tradition has protected such topics as geometric constructions, area transformations, some phases of angle measurement, and a long list of theorems which seem to point to no obvious goal. What happens, say in geometry, when we "water down" the syllabus? We omit the difficult theorems, but continue to demand that pupils solve problems which depend on them. We thus introduce a new category of "assumed theorems,"

and this contradicts the very concept of a logical system—presumably a major objective in the teaching of plane geometry. By merely attempting to simplify, we render impossible the achievement of our original objectives. Similarly, in algebra, we teach processes which are not applied in their ultimate technical purposes. Thus we add algebraic fractions in watered down syllabuses, with no real purpose or application.

When objectives are clear, then we may set up standards. If college preparation is the objective, then achievement in problem solving will determine standards. But if appreciations are the major objectives, then those standards no longer apply. Teachers who bemoan the "lowering of standards" are correct in their implied criticism of the watering down of syllabuses, but err when they apply the same complaint to a syllabus with different objectives.

It would seem from the foregoing considerations that a syllabus for our group of students must be built from a fresh beginning, with the needs of these pupils in constant view. In this manner must a teacher of the second track view his assignment. It is a serious error to teach these pupils using the traditional examination as criteria for lesson development and drill. We have been doing that for more than 15 years, with a resultant progressive deterioration in learning situations, pupil-teacher relationships, pupil attitudes toward mathematics and the school, and in "results obtained." It is vital for successful teaching of these pupils that the watering down of old syllabuses be abandoned. And it is equally vital that our preconceived notions of standards be replaced by a new set of standards which will reflect more valid objectives. We should be able to say, after teaching these pupils, that their attitude toward mathematics is less antagonistic and more appreciative; that they understand, within their limits of comprehension, the vital role which mathematics plays in our civilization; that their number concepts are functional and their arithmetic competence improved; that they have seen and enjoyed some of the recreational possibilities of mathematics; that they have gained some understanding of the general methods of mathematics; and finally, that they have learned some of the basic mathematical concepts, in terms of which a large part of our everyday thinking is done.

IV. What Kind of Teacher for the Second Track?

Since it is estimated that thirty percent of our student body belong in second track classes, the policy of assigning these classes to one or two teachers specially skilled in handling slow learners is no longer feasible. More and more teachers of mathematics will be confronted with the problem.

The question then becomes: "What characteristics and attitude of the good teachers of mathematics assume critical importance in teaching second track pupils?" And the answer lies in the areas in which second track and first track pupils show appreciable differences.

Such differences are found both in the objectives of the course and in the characteristics, capabilities, and needs of the pupils. Many of these pupils are maladjusted. Their need is for sympathy, understanding, and the hope of success. They need a teacher who is more interested in their better adjustment and relative growth than in the attainment of rigid artificial standards. If the teacher is to give the pupils this feeling of success, "to rebuild the pupil's self respect and imbue him with more favorable attitudes," he must have a strong personal conviction of the value of this learning experience for the pupil. The pupil's weakness in forming associations, his inability to generalize, his poor memory, his short attention span—all these challenge the teacher's resourcefulness, patience, and adaptability. The teacher must be willing to

evaluate achievement and the improvement of work habits with reference to progress made and not to objective measures of the attainment of a body of facts and skills. The pupil's inability to transfer his knowledge to related fields calls for a teacher who possesses a rich background in mathematical applications.

These pupils are restless and find it difficult to observe the customary routines of an orderly classroom. They fail to bring their tools, dawdle, answer out of turn, distract their neighbors and are guilty of many similar infractions. The teacher needs to keep constantly in mind that these misdemeanors are not necessarily manifestations of the extent of their maladjustment. The teacher will have made a significant contribution to the adjustment of these pupils if she has helped them to form better work habits and better attitudes.

All children need patience, kindliness, and affection. The second track pupil is lost without them.

V. Some Down-to-Earth Suggestions

While objectives, syllabuses, and materials of instruction are of vital importance, what takes place in the classroom will finally determine the success or failure of the course. The conventional procedures of the traditional classroom must be modified for the second track pupils in accordance with their special characteristic abilities, and interests.

A. Methods of Instruction and Recitation

1. Slow pupils have a shorter attention span than normal pupils. Hence, the method of recitation should be varied with periods and from period to period. The teacher should provide a variety of individual tasks, supervised study, group work, work at the blackboard, work in individual notebooks, individual and group projects, and contests. Avoid attempting to conduct a full period of the traditional question and answer type of recitation. A means of modifying the question and answer type of recitation, when used, is to require the pupils to write down answers to questions in their notebooks, or at the blackboard.
2. This type of pupil has little aptitude for dealing with abstractions and learns more easily through a concrete presentation. Hence the teacher should provide for a certain amount of experimentation and laboratory work. Materials such as oaktag, paper, and scissors interest many of these pupils. It is desirable that students shall provide their own rulers, protractors, and compasses. However, the teacher will be well advised to have a supply of spares. A system of merits may tend to induce pupils to bring their own tools. Good order in a laboratory does not necessarily mean absolute silence.
3. Another method of providing concrete presentation is through the use of concrete materials such as models, diagrams, pictures, films, and slides. Have students bring materials from home as part of a homework assignment.
4. Slow pupils have difficulty in making generalizations. Hence there should be more use of motivations, dramatizations, and various visual aids to stimulate this process. Furthermore, this lack of power of generalization results in less transfer of their knowledge to related fields. Therefore, the teacher should work with concrete situations, and apply the mathematics to practical situations, as much as possible.
5. These pupils are less resourceful mentally and seek physical outlets. They should never be left in doubt as to what is expected of them. Careful, detailed instructions for the carrying out of their tasks should be given them. A certain amount of physical movement should be provided periodically in the class, such as going to the board.

A notebook kept by each pupil for classwork provides some student activity. Medial and terminal summaries (in graphic and tabular form where possible) can be written on the blackboard and in the notebooks.

6. The slow pupil has verbal difficulties. Hence, oral reading of problems and explanations should frequently supplement silent reading. Be sure all words used are understood. Teach the use of the textbook.
7. These pupils need a sense of success. The pace of a lesson, therefore, should be such that the group can absorb the materials. Attempting to do too much or to adhere too rigidly to a set lesson plan may seriously compromise the teacher's hope of success with these pupils. Allow ample time for generalizations and conclusions. By distribution of questions and by careful inspection of work, check constantly that the pupils are learning the lesson. By using a student secretary at the board at times, the teacher may be able to get around the classroom at the same time to check individual work.
8. As slow pupils have relatively poor memories, there should be frequent reviews and drills. Use varied settings and avoid pure repetition. Practice tests graded by the pupils themselves may be helpful.
9. Flexibility in teaching procedures is of the utmost importance. The teacher must be resourceful in changing types of activities during any period should there be a loss of interest or other indication that the procedure used is not meeting with success.

B. Classroom Management

1. The details of classroom management should tend to become routinized. The pupils should have an understanding of just what is expected of them. Instructions should always be simple and detailed.
2. The assignment should be written in a specially set-apart place on the blackboard, or should be on a mimeographed sheet giving the assignments for some time ahead.
3. Some simple task or assignment can be given at the beginning of the period, to be begun while the class is getting settled.
4. The assignment of definite responsibilities to individual pupils, such as erasing blackboards, distributing or collecting materials, will help establish good routines and give the pupils a sense of participation.
5. To arouse interest and also to give the pupils a sense of achievement, attractive exhibits should be periodically posted on the bulletin board. Written work of pupils and construction work by them should be exhibited.

C. Home Work

1. Some home work should be expected of the pupils. This can impress on them that the course has its standards, just as academic courses do, and can provide them with independent activity.
2. Specific instructions should be given as to the form and quality of the homework. The homework should be simple, and within the ability of the pupils.
3. Because of the brief span of attention that these pupils have, the homework should be comparatively short, to take no more than 20 or 30 minutes. The types of homework assignments should be varied so as to include the collection of illustrative materials, verbal answers, newspaper clippings, etc, as well as the usual solution of problems or exercises.
4. Home work may be in individual units daily, or in more comprehen-

(Continued on page 546)

The Jewel and Its Setting

By Kenneth B. Henderson
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It is both educationally sound, and intellectually refreshing to back off periodically and look at the process of education. We human beings are continually exercising one of our human frailties by losing perspective. Education and the teaching of mathematics, of course, continue whether we lose perspective and direction or not. Parents pay taxes, teachers teach and draw salaries, administrators administer, and students learn a lot of things. But all are convinced that education is "a rum go" if all those interested in it miss the forest for trees.

When a person buys a diamond ring he examines both the stone and the setting. A properly cut stone in the right setting! This is a fair analogy to the situation in education. One examines the various disciplines in the secondary school curriculum in terms of the job of secondary education.

There are three points of reference one must keep in mind when considering the "oughts" of the program (content and method) of secondary education: 1. Those social processes that are basic to group living, processes which if neglected can result only in societal retrogression, disintegration, and decay; 2. The deveopmental needs of youth which become manifested as youth come of age in our confused, industrial culture; and 3. The values which uniquely characterize what we call "the democratic way of life." A sound educational program neglects none of these whether it uses the particular names given above or not. Let us look briefly at each of these, for the role of every subject in the curriculum (and this includes mathematics) should be determined in part by these points of reference.

The Basic Social Processes

When one has centrifuged the relevant research of sociologists, historians, and philosophers, there remain something like eleven basic social processes. It is not difficult to identify them. Look at how man spends his waking hours. Consider the processes that pervade, in one form or another, all ways of life. Think of the processes which when not adequately carried out result in social ulcers like juvenile delinquency, crime, unemployment, epidemics, class cleavages, divorces, wars, and mass migrations of people.

These eleven basic social processes are:

1. Producing goods and developing services
2. Buying and consuming goods and services
3. Making a home and rearing the young
4. Protecting physical and mental health
5. Spending leisure time
6. Engaging in social and civic activities
7. Educating both old and young
8. Conserving natural and human resources
9. Expressing aesthetic and religious impulses
10. Communicating with others
11. Deciding on the "oughts" of living

The on-coming crop of youth must be so educated as to be intellectually able and emotionally disposed to solve the problems associated with these processes. Any other alternative means our culture, like those of the Incas, Babylonians, Romans, and Carthaginians, will be marked "F" and consigned to the waste basket of time.

The Developmental Needs of Adolescents

For these needs, we listen to sociologists, psychiatrists and clinical psychologists, parents and the adolescents themselves. Being the kind of human "critters" adolescents are and living in the confused, industrial society in which they do, they have certain psychological-social needs. It is not hard to reach a consensus on the following:

1. The psychic hungers, e.g., feelings of se-

curity, of success, of being accepted by and belonging to a group, and of having some freedom in one's choices and actions
2. Adjustment to a changing body
3. Wholesome relationships with age-mates of both sexes and with adults
4. Gradual assumption of adult status

Of this we are sure—if these needs are not adequately met, our schools sow wheat and reap chaff, and several of the basic social processes mentioned above are impaired.

The Values Which Define the Democratic Way of Life

The basic social processes exist in every society, but how they are carried out depends on the "oughts" accepted by the society. These processes enable one to make a choice among alternative courses of action; to do certain things and refrain from doing certain others. These "oughts" emerge by implication from the postulates to which we are committed. They are:

1. All men are of supreme and hence of equivalent moral worth.
2. Given the facts, the common man can be trusted to arrive at intelligent solutions to the problems associated with the basic social processes listed above.
3. The welfare of all is promoted as each individual accepts this as his responsibility, and works cooperatively with others in the solution of the problems of living.

To these three sets of tasks, the subjects of the curriculum must face up. Of mathematics we specifically ask: What does it have to offer? By nature it is a language—a basis of communication when man thinks in terms of quantitative, form and space relations. By nature it is a method of thinking as well as a body of organized implications. How does the jewel fit the setting?

Mathematics and the Basic Social Processes

Every significant social problem has a quantitative aspect to it. As a matter of fact, we do not begin to get excited about most problems save as we think in terms of quantitative data. The A-bomb scares us so because only one—and an old-fashioned one—killed somewhere near 100,000 people.

For people to cooperate in the solution of problems associated with social processes, they must be able to understand each other. No communication; no cooperation. Hence they must be able to read, speak, and write the language of mathematics. And one does not have to wait until students become adults before this need is real.

It seems equally clear that people should have a command of those mathematical operations and processes which must be understood in order to make computations with exact and approximate numbers. This is clearly a responsibility of mathematics, and, from what experimental evidence we have, cannot be taught incidentally. As Fawcett has pointed out,[1] such a mastery probably cannot be attained if mathematics courses are absorbed by a core or common learnings course, as worthwhile as such a course might be in the setting described at the outset. Most general educators recognize this and provide in their "picture-in-the-head" of the curriculum elective courses in mathematics outside the core.

There is a third implication. Youth and adults alike should understand the technique of solving problems, especially the technique which involves several people working together. Subsumed under this are such skills as stating problems which offer a solution; formulating fruitful hypotheses; gathering, organizing, interpreting, and evaluating data; drawing inferences; examining the inferences of others; and maintaining good human relations when several people work together.

To be sure, it cannot be said that such skills can be taught only or even chiefly in mathematics classes. They are "stuff for the troops" in every subject. The point is that mathematics teachers can tailor the subject to provide an understanding

[1] H. P. Fawcett, "Mathematics and the Core Curriculum," The Mathematics Teacher, XLII (Jan., 1949), 6–13.

of, and practice in using these skills. This will probably mean somewhat of a shift in emphasis and in the kinds of experiences provided students in the typical mathematics classroom. This can be done and still provide a proper balance between this and teaching students the operations of arithmetic and algebra.

There is a need to study significant problems related to the basic social processes discussed above. Such a study should focus on those problems in which quantitative relationships or the facts and relations of form and space predominate. There are problems of the finances of earning and spending one's money; of getting one's money's worth; of various forms of credit, investment, and protection against risks; of interpreting tables, graphs, and other expressions of functional relations; of deciding whether the town needs a new school or hospital; of polling public opinion on some civic or school issue; of how the government raises and spends money; of measuring the run-off water from nearby fields; and so on.

Some mathematics teachers—judging from what they do and say—would as soon be seen leaving a burlesque show as be found discussing social problems or issues in a mathematics class. Yet if we are to help students clarify social goals and decide about the "oughts" of living we cannot leave this entirely to the social studies teacher.

Values are best clarified in the context of real problems. A teacher whose class is studying the cost of national health insurance as against "every man for himself" misses the boat if the "ought" of such a plan is not considered. Certainly this involves subjective and non-quantitative data! But it also involves thinking—intentional, critical, and reflective thinking. There will be stated and unstated assumptions, accurate and inaccurate data, sound and unsound reasoning, good and bad authorities, and hence opportunities to examine all of these. The mathematics teacher ought to be able to help students in both spheres: to clarify their ideas concerning the "oughts" of social living, and to sharpen their ability to think as logically and consistently in this matter as in geometry for example.

There is another clear implication. To aid in the solution of our problems of living we need specialists who will provide the leadership and supply the "know how." Particularly, we need engineers, statisticians, doctors, top management personnel, teachers, research workers, and a host of others. Many of these will need all the mathematics time will allow them to get. In training the future specialized personnel, the role of sequential mathematics courses in high school is a vital one.

Mathematics and the Needs of Adolescents

Mathematics does not offer a bare larder for meeting this set of needs. It has been pointed out how concerned youngsters are about whether their physical development is proceeding normally. An answer to this question is through comparisons and measurements which the students can make. An understanding of concepts like "average" and "dispersion" helps them answer their questions. The interpretations they make from tables and graphs supplied by a resourceful teacher will provide further help. But without adequate mathematical understanding and instruction in interpreting data, they will not get the help they need.

Let us look at the psychic hungers of human beings, adolescents included. One is the need for a feeling of security. Now a feeling of security, in part, is derived from feelings of being able to cope adequately with a situation. Suppose a person is relatively incompetent in ordinary arithmetic. Percentage is very nebulous. Fractions give him an uneasy feeling. Even a simple formula is an enigma. Such a person will find it difficult to maintain his ego-ideal when faced with situations demanding mathematical proficiency. Doubting his competency, he may become

fearful and lacking (in such situations) in the self-confidence that feeds a feeling of security.

In the past some educators have indicted mathematics for destroying feelings of self-confidence and security because of the failure of students to succeed in mastering the subject. This indictment is a fair one, but can be easily met by smaller classes, proper teaching, etc. But many educators, parents, and students have failed to realize that the proposed remedy—*eliminate mathematics,*—is certain to engender feelings of insecurity. One cannot dodge the fact that a person must be mathematically literate to live in the world today and in the world he will face in the future. Building secure children means equipping them with the tools it takes to get along in this life, a life becoming increasingly mathematical.

Our belief in the dignity of every child makes us believe every child should experience success in some way. There is a story for mathematics in this. We are not clear what mentality is, but we know that some children are born "long" in what it takes to succeed in abstract thinking. They are the ones who "eat up mathematics." If for no other reason than just because a student is successful and gets a "kick" out of manipulating mathematical concepts and relations, he should be permitted to have such experiences. Suffer some children to dig deeply into mathematics, for such is the joy of living and the source of future pleasure in leisure time activities.

In meeting the students' needs for acceptance by their group and assuming increasing responsibility for managing their own affairs, the mathematics teacher is no different from any other teacher. If he considers these needs important, he will plan his teaching so as to include activities involving small groups which plan their work largely by themselves. He may plan with the class how a unit should be studied, how much time should be devoted to the study, and how their work should be evaluated. In short, he does not make all the decisions; he teaches students to make decisions.

To draw the isolates into the group, the teacher can plan activities in which these folks probably will do well. Identification of their special abilities such as those associated with woodwork, making posters, willingness to collect various kinds of data, etc. may lead to their being sought after by the other students. If acceptance is withheld because of undesirable personality traits such as being unclean or insincere, a personal talk in many cases can straighten out the situation. In this sense the mathematics teacher functions as a friendly counselor as does every teacher at times.

Mathematics and Democratic Values

Our final consideration of the place of mathematics in the setting of secondary education should be its contribution to underwriting an acceptance of some of the postulates of democratic living. This aspect of mathematics teaching is often missed by teachers who spend ninety-five per cent of their time drilling their students on manipulations. A study of the lives of some of the famous mathematicians, some Jewish, some Russian, some eccentric, many poor, ought, if properly handled, develop the empathy which is basic to tolerance as well as engender a reverence for the potentialities of the human mind.

Intolerance is seated, as we know, in ignorance. It may be just as deeply seated where there is a belief in absolute truth. After all, if one has the answers, of what value is there to being tolerant with anyone who persists in questioning them? It is possible to portray the relativity of truth through a study of the arbitrary nature of units of measure and symbolism, or through a study of Non-Euclidean geometry. Such studies are within the maturity level of high school students. I have seen dogmatism and smugness in some social

beliefs of students disappear (at least overtly) when the principle was extended from mathematics to the realm of social beliefs.

We say democracy rises or falls with the ability of the common man to think—creatively, independently, and critically. The contribution of mathematics and particularly of geometry to developing such understanding and skill is well known. We do not want for practicable suggestions; we want more for more teachers who will use the method of thinking in geometry as the center of attention. We should be turning out more students who appreciate the significance of words and their impact on thinking, the importance of clearly defined terms whether defined formally or operationally, the role of assumptions in thinking, inductive as well as deductive proof, the weight of evidence, and the dependence of conclusions on assumptions and the rigor of the reasoning. While the mandate to teach students how to think is clear, more energy is expended in high schools in teaching them what to think.

And Finally

It is obvious that the sparkle of the jewel of mathematics is intrinsic. It needs only to be cut so as to bring out the sparkle. In less figurative language, the educative experiences mathematics has to offer are a function both of the "what to teach" and the "how to teach it." The key is to keep an eye on the tasks of secondary education, and let the sparkle emanate from this setting.

Teaching Second Track Mathematics

(*Continued from page 541*)

sive units for longer periods of time. While homework should be the rule rather than the exception, there should be at least one day in the week on which no homework is required.

5. To help establish a proper respect for good workmanship, the homework should be checked. This will also tend to emphasize the importance of regular performance of tasks. The homework should be marked periodically, and the mark should be included in the term mark.
6. A good technique is to require that all homework be kept in a notebook which should be checked regularly by the teacher.

D. Evaluation

1. Written tests should be given frequently. There should be both short and long tests. They should be simple, for success on these tests will help bolster self-confidence. They will also reveal what remedial work is needed.
2. Specific instructions for taking the test should be given. Expected arrangement of the work should be discussed.
3. Questions should be clearly stated.
4. Material tested should be comparatively recent, as these pupils have short memories.
5. Frequently, notebooks and textbooks should be permitted as reference materials. Formulas to be used should be supplied in many instances.
6. Allow sufficient time for completion. Make adjustments if the test has proved too long.
7. To encourage these pupils, many of whom have previously experienced failure, emphasize progress shown. Post excellent papers. Students can be encouraged to keep their own progress charts or graphs.
8. Final grades should be based upon effort and industry as well as actual achievement. In general they should be a measure of the pupil's progress in relation to himself. Complete failure should be a relatively rare occurrence.

Let's Teach Angle Trisection

By B. E. MESERVE
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EVERY YEAR several people announce that they have discovered a method for trisecting angles in the belief that they have solved the classical trisection problem. Many of the methods discovered are correct in the sense that they may be used to trisect an arbitrary angle; many of the methods have been discovered so frequently that they are practically classics themselves. The part of the problem that these discoverers apparently do not understand is that the classical problem requires that the trisection be done using only compasses and a straightedge. The straightedge may be used only to draw straight lines. In solving the classical construction problems one is not allowed to use the length or width of the straightedge, and similarly one is not allowed to use marks upon it. When these classical restrictions upon the trisection problem are disregarded, many of the constructions for the trisection of angles can be performed using only the methods now taught in secondary schools. The restriction that there should be no marks upon the straightedge is disregarded in one of the simplest and oldest constructions of the trisection of angles. This construction is attributed to Archimedes. It requires only compasses and a straightedge with two marks on it. A ruler will serve very well.

Given any angle $\sphericalangle ABC$ (Figure 1) construct a circle of radius r about the vertex B. Let D be the intersection of the circle and the side BC. Extend the side AB through B. On the straightedge mark off the length $r=BD$. Now keep the straightedge on D and slide one mark along AB extended until the other mark contacts the circle at some point F. Draw DF and let it intersect AB at E. Then $\sphericalangle BEF = \frac{1}{3}\sphericalangle ABC$. This fact may be proved as follows: Draw BF and designate the angles as in Figure 1. Then $EF=FB=BD=r$, $\sphericalangle 1=\sphericalangle 2$, $\sphericalangle 3=\sphericalangle 4$. Since $\sphericalangle 3$ is an exterior angle of triangle BEF and $\sphericalangle 5$ is an exterior angle of triangle BED, we have $\sphericalangle 3=\sphericalangle 1+\sphericalangle 2=\sphericalangle 1+\sphericalangle 1$, $\sphericalangle 5=\sphericalangle 1+\sphericalangle 4=\sphericalangle 1+\sphericalangle 3=\sphericalangle 1+\sphericalangle 1+\sphericalangle 1$ and thus $\sphericalangle ABC=3\sphericalangle BEF$. This is not a solution of the classical trisection problem since marks are used on the straightedge.

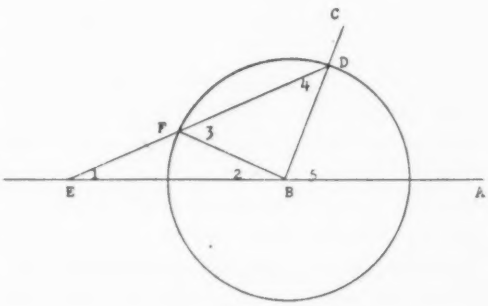

FIG. 1

There are also numerous mechanical angle trisectors that disregard all the classical restrictions upon the trisection problem. One of the best known was designed by a London barrister, Alfred Bray Kempe (Plate 1). It is based upon similar contraparallelograms and is described by Joseph Hilsenrath in THE MATHEMATICS TEACHER, October 1937, pages 282–283. Another trisector was designed by James

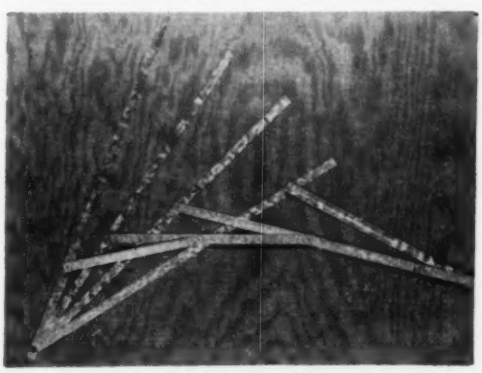

PLATE 1

Joseph Sylvester (Figure 2). Since this type of mechanism could be used to divide an angle into any number of equal parts, it is called an "isoklinostat." The origin of Sylvester's basic idea for this linkage may be indicated by the following quotation from the title of the paper in which it was first published—"On a Lady's Fan. . . ".

Mathematical literature contains a great deal of material on the trisection of angles.

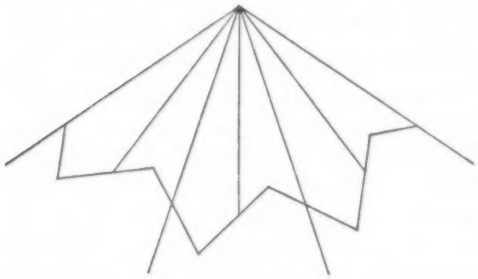

FIG. 2

Yates' book, *The Trisection Problem*[1] contains descriptions of seven curves and fifteen mechanical devices, including the two mentioned above, that may be used in trisecting angles. Periodicals frequently contain material that could be used in class on this topic. Several methods of trisecting angles are discussed in *Civil Engineering*, March 1948, pages 50–51. A method developed by a high school student may be found in THE MATHEMATICS TEACHER, October 1950, pages 278–279. More technical discussions involving the impossibility of the classical problem may be found in many college textbooks in the theory of equations.

We have now observed that (1) the trisection problem has become a game with many people who are not adequately informed of the nature of the classical problem; (2) the construction of mechanical trisectors and the trisection of any angle may be performed in many secondary school classes without introducing any new methods or ideas; and (3) there is a great deal of literature on the subject for any interested teacher or student. It is the thesis of this paper that proper instruction in the nature and types of possible solution of the trisection problem are a part of the social responsibility of the teachers of mathematics. Let us now examine the problem in detail and determine its nature.

The trisections with straightedge and compasses of certain special angles such as 90° and 180° are regularly taught in most plane geometry courses. The classical trisection problem requires a method that may be applied to *any* given angle. This problem will therefore be impossible if there exists a single angle that can not be trisected in the prescribed manner. Before specifying one particular angle that can not be trisected using the classical methods, let us consider briefly the limitations of these methods from an algebraic viewpoint.

Using a straightedge we may draw a line through any two given points. Using a pair of compasses we may draw a circle with any given point as center and any given distance as radius. Using both the straightedge and the compasses we may find the intersection of two lines, the intersections of a line and a circle, and the intersections of two circles whenever such exist. From an algebraic viewpoint any line may be considered as the graph of a linear equation, any circle as the graph of a quadratic equation. The determination of the above intersections then corresponds to the simultaneous solution of two linear equations, a linear and a quadratic equation, two quadratic equations respectively. Using these processes we may add, subtract, multiply, and divide the numbers represented by any given segments, i.e., we may perform the four rational operations graphically. We may also extract the square root of a number represented as a segment. However, these five operations are the only ones that can be performed using straightedge and compasses in the sense that an operation can be performed

[1] Robert C. Yates, *The Trisection Problem* Published by the author, (West Point, N. Y., 1942).

in this classical manner if and only if it can be accomplished by a finite number of the above five operations.

Given any unit of distance we may construct a segment of length d if and only if the number d may be represented in terms of the unit 1 using the four rational operations and the extraction of square roots a finite number of times. Thus all rational numbers and expressions such as $\sqrt{10-2\sqrt{5}}$ may be constructed using straightedge and compasses as soon as a unit is known. The above expression represents the length of a side of a regular pentagon inscribed in a circle of radius two. Such numbers and distances are called constructible. With respect to a given coordinate system a point may be constructed according to the classical methods if and only if its coordinates are constructible. A line may be constructed if and only if it may be expressed by a linear equation with constructible coefficients. In general, *a geometric figure is constructible using straightedge and compasses if and only if the coordinates of its points (vertices, etc.) can be constructed from those of the given figure using a finite number of rational operations and extractions of real square roots.* It is not sufficient to be able to construct a line as close to the desired line as one wishes. One must be able to draw the desired line.

The usual proof that there exists at least one angle that can not be trisected using the classical methods makes use of a few trigonometric identities and an angle of 120°. Given an angle of 120° we seek an angle of 40°. The construction of an angle of 40° is equivalent to the construction of a right triangle with hypotenuse unity and base cos 40°. Thus an angle of 120° can be trisected if and only if a segment of length cos 40° can be constructed. From $\cos 120° = -\frac{1}{2}$ and the trigonometric identity

$$4\cos^3 x - 3\cos x = \cos 3x$$

we may obtain

$$4\cos^3 40° - 3\cos 40° + \tfrac{1}{2} = 0.$$

Taking $y = \cos 40°$ we have

$$4y^3 - 3y + \tfrac{1}{2} = 0.$$

Multiplication by 2 and the substitution $z = 2y$ now gives

$$z^3 - 3z + 1 = 0.$$

The only possible rational roots of this equation must be integers since the leading coefficient is unity and must divide the constant term. Thus the only possible rational roots are $+1$ and -1. Since neither of these integers is a root, the equation has no rational roots. If the equation had a quadratic surd root $a+b\sqrt{2}$, it would also have the conjugate quadratic surd $a-b\sqrt{2}$ as a root, and the remaining (third) root would be rational. Since any constructible number may be expressed using a finite number of rational operations and extractions of real square roots, every non-rational constructible number is one of a set of an even number of conjugates (including itself). Thus any equation with integral coefficients having a constructible surd root must have an even number of surd roots. In particular any cubic equation with integral coefficients having at least one constructible surd root must have exactly two surd roots and one rational root. Thus the fact that the above equation has no rational root implies that it has no constructible root. Since z is not constructible and $z = 2\cos 40°$, we have proved that cos 40° is not constructible and the angle of 120° can not be trisected using straightedge and compasses. Thus the classical trisection problem is impossible.

Two other classical construction problems may be quickly disposed of on the basis of the above discussion. It is not possible using straightedge and compasses to construct a cube with a volume double that of a given cube (Delian problem) since this requires the construction of a segment of length x where $x^3 = 2$ or $x = \sqrt[3]{2}$ taking the side of the given cube as unity nor is it possible to construct a square

with area equal to that of a given circle.

Models of mechanical angle trisectors will catch the eye of almost any secondary school student. Archimedes' construction for the trisection of any given angle less than 90° may be given at almost any time in secondary school. The proof of this construction may be considered as soon as the students recognize that an exterior angle of a triangle is equal to the sum of the opposite interior angles. Since an angle of 30° may be constructed, the students may be shown how Archimedes' construction may be used to trisect any given angle by first subtracting an angle that is a multiple of 90°. Even the above demonstration that the classical trisection problem can not be solved may be considered after the student has studied a few elementary trigonometric identities and advanced algebra in secondary school. The whole question of the trisection of angles may also serve as a basis for a very enjoyable meeting of a mathematics club.

We have now seen that not every angle may be trisected by the classical methods and therefore the classical problem of trisecting an angle, i.e., trisecting an angle using only straightedge and compasses, is impossible. We have also seen that there exist simple methods for trisecting angles using common instruments, such as a ruler or even a protractor, that are forbidden under the classical restrictions of the problem. The frequent "solutions" published in newspapers or even in copyrighted pamphlets published at considerable expense to the discoverer are a constant reminder of the failure of the public schools and colleges in the past to inform the students properly regarding this very interesting problem. The popularity of the problem may in part be due to the fact that it is often stated to be impossible without a full explanation of the reason for this verdict or the classical restrictions that make it necessary. Although this cloud of confusion can not be dispersed suddenly by any path of action, it could be gradually cleared away by the acceptance of all mathematics teachers of a responsibility to see that the students are correctly informed as to the nature and types of possible solution of this most interesting problem, the trisection problem. Let's teach angle trisection.

Editor's Note

The editor expresses his appreciation to the following persons, in addition to the associate editors and the editors of the various departments, who have refereed papers and rendered other valuable assistance in the publication of THE MATHEMATICS TEACHER during the past year:

M. H. Ahrendt, Frank Allen, H. Glenn Ayre, William Betz, Nanette R. Blackiston, Carl B. Boyer, Robert L. Burch, Frances Burns, C. H. Butler, Walter Carnahan, Dale Carpenter, J. W. Cell, Paul Clifford, Wayne Dancer, David Davis, W. E. Edington, Harold Fawcett, Irwin Feinstein, J. S. Frame, H. E. Grime, Fred Gruenberger, Lucy Hall, M. L. Hartung, Ida Mae Heard, Gertrude Hendrix, Lesta Hoel, T. C. Holyoke, Lenore John, Houston T. Karnes, Mildred Keiffer, Rachel Keniston, Ona Kraft, Florence Krieger, Ruth Lane, Herbert L. Lee, Eunice Lewis, Virgil Mallory, Bruce Meserve, O. E. Overn, Philip Peak, Homer C. Peterson, Emma N. Plank, A. W. Recht, C. C. Richtmeyer, J. B. Rosenbach, M. F. Rosskopf, Elizabeth Roudebush, Robert E. K. Rourke, John Satterly, Irene Sauble, Veryl Schult, Carl Shuster, Jessie Smith, Ben A. Sueltz, Louis Thiele, Gilbert Ulmer, J. Fred Weaver, Kenneth W. Wegner, Marie S. Wilcox, F. Lynwood Wren, and Robert Yates.

Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers*

By W. I. Layton

Stephen F. Austin State College, Nacogdoches, Texas

This paper will attempt to show the mathematical training prescribed by eighty-five institutions of higher learning which prepare elementary teachers. These institutions are all accredited by the American Association of Colleges for Teacher Education. Geographically speaking, they are located in forty-five states and include state colleges, state teachers colleges, state universities, and privately endowed institutions.

Catalogs available as of June 19, 1950 formed the basis for this study and consequently all of the material referred to is of a recent nature.

We shall investigate the following: mathematics required or recommended for admission to the freshman class of institutions which train elementary teachers; courses in mathematics content required for elementary teachers in the one-, two-, three-, four-, and five-year courses of study; courses in mathematics methods required for elementary teachers in the one-, two-, three-, four-, and five-year curricula; and, for purposes of comparison, requirements in art, English, and geography content in the one-, two-, three-, four-, and five-year programs of study for elementary teachers.

Let us examine now the college entrance requirements in mathematics for those preparing to teach in the elementary school. We refer to the unconditional admission of students on the basis of high school units required. We do not refer to admission through examination or to the admission of mature students. We assume that mathematics is not required for admission to the colleges whose bulletins were considered unless it is specifically referred to as an entrance requirement.

Of the eighty-five catalogs studied, three gave insufficient data concerning admission. Of the remaining eight-two, twenty, or approximately one-fourth, specify mathematics as a requirement for admission. For these twenty schools which require mathematics for entrance the mean is 1.20 high school units.

A few of the catalogs prescribe certain mathematics courses for admission. Four name at least one unit of algebra as a requirement, and one specifies geometry in addition to algebra. One school requires one unit in algebra and one unit in geometry for mathematics and science majors and minors.

Eleven colleges recommend mathematics for admission to the freshman class of their teacher training programs. The mean of these recommended units is 1.82. One catalog advises two units in algebra and one and one-half units in geometry for mathematics and science majors and minors. Another catalog suggests that the student present one unit in algebra and one unit in geometry if he plans upper grade teaching, and one and one-half units of each for a teaching field in mathematics.

Several special cases of admission seem pertinent. One college stipulates that the student must take a three semester hour college course in elementary algebra and a three semester hour course in plane geometry unless he has presented one high school unit in each of these subjects for college entrance. Another institution

* Based upon a paper presented at the Eleventh Christmas Meeting of The National Council of Teachers of Mathematics at Gainesville, Florida, December 27–29, 1950. Some related studies were discussed in the original paper which are not included in this more abbreviated version.

has a similar requirement except that the student must have earned a grade of C or better on the final semester of each high school course. College credit will be granted on the college elementary algebra and plane geometry courses unless the corresponding high school course was completed with a grade of C or better in which case the repeated course will be classified as a "refresher" course carrying no college credit. Students who have one semester of high school credit in trigonometry, intermediate algebra or solid geometry may be permitted to repeat the corresponding course in college for one and one-half units of college credit.

An admission practice described in another catalog states that the student who does not have the one unit in algebra, geometry or trigonometry required for entrance must make up the deficiency by enrolling in non-credit courses before full admission is granted.

The last special case to be mentioned is that of a college which requires no mathematics for admission. However, students who will elect mathematics in college must present three high school units of mathematics of which algebra and plane geometry must have been taken in the senior high school.

Closely related to the problem of admissions is the practice of giving proficiency tests in the fields generally taught in the elementary school. Each of the following practices described is peculiar to one institution:

(1) Students when applying for admission to the teacher training program must display competence in the use of the English language, in arithmetic, handwriting, reading and spelling, indicated by having passed the "Fundamentals Test." The college offers non-credit courses for those who fail to qualify on the test.

(2) All students who seek the kindergarten-primary, general elementary, or junior high credential must stand an examination in arithmetic. Students who fail this test may prepare for a retest by self study, private coaching, or a sub-collegiate arithmetic course.

(3) The student must either display sufficient proficiency in arithmetic on the basis of a test or take a course in fundamentals of mathematics as a basis for further required mathematics.

(4) All students showing a deficiency on the pre-enrollment mathematics test must take a non-credit course in arithmetic.

(5) Entrance examinations are given in English, reading, American history and problems of democracy, general mathematics, general science, and speech.

(6) Preliminary non-credit courses in art and music are arranged for students who are not prepared to carry the credit courses in these fields without further instruction.

(7) The student may be required to take preliminary non-credit courses in art unless satisfying scores are made on qualifying tests.

(8) All students must pass an examination on the subject matter of arithmetic.

(9) Those who do not make a satisfactory showing on the entrance test in English are required to take a non-credit course in fundamentals of English.

(10) Upper elementary students must prove their proficiency in the fundamentals of mathematics for the upper grades. Those failing to pass a proficiency test must take a non-credit course in arithmetic for teachers. Any student who has successfully completed a college course in mathematics is excused from the test.

The major portion of this paper is concerned with the mathematical training prescribed by colleges in the preparation of elementary teachers. A majority of the eighty-five colleges studied offer the four-year curriculum for elementary teachers. It is interesting to notice in Table 1 that the mean of the mathematics content requirement in the various four-year curricula is 1.63 semester hours, while the means for art, English and geograpyh content are, respectively, 4.28 ,11.46, and 4.30 semester hours. Mathematics stands at

the foot of the list. The requirement in art and also in geography is more than two and one-half times that in mathematics. The mean in English is approximately seven times the mean in mathematics.[1] However, thirty-five schools have a requirement of one or more courses in mathematics content in their four-year programs of study. This requirement ranges from zero to eight semester hours. We must realize in this connection that a number of the eighty-five colleges studied require courses which are partly content and partly methods in mathematics. In fact, twenty-eight colleges require courses of this nature. The range for these part content-method courses is from two to four semester hours. A total of sixty-three of the colleges require some mathematics content in their four-year curricula either in the form of purely content courses or in connection with courses which are partially method. It is doubtful, however, that the content mathematics treated as part of a methods course receives the serious attention it merits.

Turning now to the mathematics methods required we learn from Table 1 that the mean is .47 of a semester hour for all colleges having four-year curricula. Twenty-six colleges require courses in mathematics methods. Sixteen institutions require training in mathematics methods as a portion of a methods block which includes various other elementary school subjects. Typical courses along this line include methods in reading, language, and arithmetic, or in language, social studies, and arithmetic, or possibly in arithmetic and science. Counting the twenty-eight schools already mentioned as requiring mathematics courses which are part content—part method, we arrive at a total of seventy colleges which call for some contact with mathematics methods on their four-year curricula.

Ranking second among the curricula most frequently offered is the two-year program for training elementary teachers. According to Table 1, the mean of mathematics content required by all institutions having two-year curricula is .54 of a semester hour. The corresponding means for art, English, and geography are 2.84, 7.08, and 2.55 semester hours. Again mathematics holds the low position as far as requirements are concerned. Its mean is only about one-fifth as large as that of art and also of geography, while the mean of English is thirteen times as great as the mean in mathematics. Of the twenty-five schools having two-year programs only six require at least one course in content mathematics with a range from zero to three semester hours. Nine schools specify courses combining mathematics methods with content. Thus fifteen institutions call for some kind of training in content mathematics on the two-year programs.

Only six schools which have two-year curricula for elementary teachers require methods courses in mathematics. As is shown in Table 1, the mean requirement in mathematics methods for all two-year curricula is .39 of a semester hour. Four colleges require courses combining methods in several elementary subjects and nine treat mathematics content and

TABLE 1

*Means in Semester Hours of Certain Courses Required in Curricula for Elementary Teachers**

Curriculum	Mathematics Content	Methods in Mathematics	Art Content	English Content	Geography Content
4 year	1.63	.47	4.28	11.46	4.30
2 year	.54	.39	2.84	7.08	2.55

* Courses which are part content and part method are not included in this table.

[1] Speech and dramatics are not included in the English content requirements.

method in the same course. Therefore, nineteen of the twenty-five colleges with two-year curricula specify mathematics methods in their programs.

Of the eight schools which have three-year curricula for training elementary teachers, six require mathematics content. This requirement ranges from the content included in a two-hour combined content-methods course to four semester hours of pure content. No courses which deal strictly with mathematics methods are required for the three-year curricula, but five colleges call for two-, three-, or four-hour courses which are part content and part method.

Seven institutions list one-year training programs most of which are designed for the preparation of rural elementary school teachers. Only one of these curricula requires mathematics content. Two catalogs specify methods in mathematics—one and one-third and two and two-thirds semester hours, and another requires a methods course which may include mathematics along with other subjects. One college calls for a two and two-thirds semester hour course in mathematics content and methods combined.

Of the three schools which have two and one-half year curricula, one calls for three semester hours in content mathematics and another for an unspecified amount in that subject. None of these mentions mathematics methods.

Two schools have plans calling for two and one-third years or seven quarters of training for elementary teachers. While one of these has no requirements pertaining to mathematics, the other specifies two and two-thirds semester hours in content and one and one-third hours in mathematics methods.

One college has a six-week program, one a twelve-week, and one a one and one-third year plan for those who will teach on the elementary level. None of these has any type of mathematics requirements. The six-week program is to be discontinued after September 1, 1952.

Of special interest in this paper is the five-year program leading to the master's degree in elementary education. Of the eighty-five colleges studied, twenty which offer a master's degree in this field furnished sufficient data for evaluation. Not one of these schools specifies any courses in mathematics content or methods in its fifth year. This is shown in Table 2. Three of these twenty just referred to did not furnish information concerning their undergraduate training for elementary teachers. Of the remaining seventeen, twelve colleges or 71 per cent, require no mathematics content courses in all of their four-year elementary education programs. Of the same seventeen colleges, 65 per cent require no courses in methods in mathematics as revealed in Table 2. Frequently when it is suggested that some mathematics content be required in the curriculum for training elementary teachers the cry goes up, "There is insufficient time to crowd in all the courses we would like to include." It seems significant to observe in Table 2 that of the colleges supplying sufficient data on both their undergraduate and graduate programs, 71 per cent had no place in either their four-year or their five-year degree curricula for any

TABLE 2

Per Cent of Colleges not *Requiring Certain Courses in Curricula for Elementary Teachers**

Curriculum	Mathematics Content (%)	Methods in Mathematics (%)
Master's Degree	100	100
Four Year	71	65
Five Year	71	65

* Courses which are part content and part method are not included in this table.

mathematics content courses. This is probably the trend followed by a majority of institutions which have four- and five-year degree programs for elementary teachers.

Only one college has a requirement in mathematics methods for its master's degree. This is a two and one-half semester hour course called natural science and mathematics curriculum. Since this course does not deal solely with methods in mathematics, it is not included in Table 2.

Let us now examine the titles of the various mathematics content courses required in one or more of the curricula for those who will teach in the elementary school. Ten of the colleges specify general mathematics and three arithmetic for teachers. The remaining titles which will be listed below are each required by one college. These titles are: applied mathematics, fundamental mathematics, socialized mathematics, basic mathematics, practical mathematics, remedial and functional mathematics, background of mathematics, introductory course in mathematics, introduction to college mathematics, mathematics in the upper grades, mathematics for elementary teachers, mathematics for elementary grades, foundations in arithmetic, fundamentals of arithmetic, curriculum in arithmetic, college arithmetic, arithmetic, arithmetic in modern life, college algebra or college arithmetic, college algebra or basic mathematics, college algebra and/or trigonometry or business mathematics. Thus, college algebra is never required outright but may be taken as an alternate with one or more other courses. One of the courses in this list, entitled introduction to college mathematics, perhaps deserves special mention inasmuch as the content of the course varies according to the previous preparation of the students.

It is interesting to note that one college offers a fine arts four-year program which does not include specialization in music. This fine arts curriculum prepares for teaching fine arts in the elementary schools and in junior and senior high schools. The study revealed no specialized curriculum for prospective elementary school teachers of mathematics.

Two catalogs, in describing the requirements for a certificate valid in both the elementary and secondary schools, call for a minimum of twenty-four semester hours in each of two teaching fields with the exception of the social sciences and mathematics. These fields require thirty-two and twenty-two semester hours respectively. Here again, mathematics seems to be discriminated against with respect to the other subject fields. In one of these schools two semester hours may be deducted for each high school unit in mathematics, the maximum deduction to be six hours. This practice would seem to weaken further the college training in mathematics received by those who take advantage of the allowable deductions referred to above.

Summary

Only one-fourth of the catalogs studied specify mathematics as a requirement for college entrance for freshmen who enter with a teacher training objective. This seems to indicate a weak foundation for our potential teachers of mathematics in the elementary school as far as admission requirements are concerned.

In the four-year curricula, the requirement in content art and also in geography is more than two and one-half times that in mathematics. The mean in English is approximately seven times the mean in mathematics content.

In the two-year programs, the content mathematics requirement is only about one-fifth as large as that of art and also of geography, while the mean of English is thirteen times as great as the mean in mathematics content.

In both the two- and four-year curricula mathematics stands far below the other content subjects considered. A nation-wide study of certification requirements for teachers of mathematics was made by

the writer and published in 1949.[2] Some of the results contained in this book were later published in THE MATHEMATICS TEACHER.[3] The findings of the present investigation parallel those of the certification study in which mathematics stood at the foot of the list in terms of the requirements for certifying those who teach mathematics in the elementary school. At the time the results of the certification study were made public there were those who commented, "But the colleges provide far better programs for the training of teachers of mathematics in the elementary school than the state certification laws prescribe. Actually, mathematics is not slighted by the colleges nearly as much as your certification study would indicate." The means computed in the present study and those in the certification study are not directly comparable, but according to the present study, mathematics is still at the foot of the list of the required content subjects considered—English, geography, art, and mathematics. In fact, there is quite a big gap between the mathematics content requirement and that in the other subjects. This inadequate preparation in mathematics for teachers in the elementary school is an alarming situation not only in view of the over-all future of our civilization, but even more especially now with the increased demands placed upon mathematics due to the present national emergency. It would seem that this mathematical crisis is not going to be corrected until strong organizations like The National Council of Teachers of Mathematics and The Mathematical Association of America together with the state organizations of mathematics teachers put their shoulders to the wheel and act through the state certification channels to improve this pitiful situation in which mathematics finds itself. Probably the colleges will not improve their standards in mathematics for the training of elementary teachers until the state certification laws require them to do so.

On the basis of the present paper the graduate schools considered which offer degrees in elementary education do not require mathematics content in their fifth year of training, and the majority of these same schools do not require mathematics content courses in their four-year programs. It seems very regrettable that with five years being devoted to the preparation of elementary teachers in these schools, there is still no time for required courses in content mathematics. With more and more states looking toward a five-year program for training elementary teachers, it might behoove the mathematical organizations to devote some attention to the inclusion of mathematics content in these programs.

[2] William Isaac Layton, *An Analysis of Certification Requirements for Teachers of Mathematics* (Nashville, Tennessee: Bureau of Publications, George Peabody College for Teachers, 1949).

[3] W. I. Layton, "The Certification of Teachers of Mathematics," THE MATHEMATICS TEACHER, XLII, No. 8 (1949), 377–380.

The 1951 Christmas Seal Sale of the National Tuberculosis Association will be conducted from November 19 to December 25. Christmas Seals have become a holiday tradition in this country where they have been sold since 1907. Each year 94 percent of the money raised by the sale of Christmas Seals remains in the state where it was contributed to support local and state tuberculosis control. The remaining six per cent is forwarded to the National Tuberculosis Association. At least one-sixth of this percentage is used for medical and social research.

Tuberculosis kills more people in the United States than all other infectious diseases combined. It killed approximately 40,000 persons in the United States in 1949, at the rate of 100 a day, one person every 13 minutes. Estimates place the number of tuberculosis deaths all over the world at almost 5,000,000 a year. Tuberculosis is the chief cause of death among children throughout the world, according to the United Nations International Children's Emergency Fund. In the United States, it annually kills approximately 1,600 children under 15 years of age.

Since the campaign for tuberculosis was inaugurated almost 50 years ago, the death rate has been forced down 85 per cent. This has meant a saving of 5,000,000 lives for, if TB had continued to kill at the rate of 50 years ago—194 per 100,000 population—it would have killed 5,000,000 more people than have died of tuberculosis in the intervening years.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

36. Additive and Multiplicative Magic Squares.

Miscellanea 16.[1] showed how magic squares could be used in a new version of the old game of Tick-Tack-Toe. At the end of the explanation of this game was the statement, "The connection with magic squares is also obvious and might well motivate a report or project dealing with this latter topic." The purpose of this note is to aid in the realization of this project. It is my opinion that the device of the magic square has immense possibilities in mathematics teaching, not merely for the mathematics club where it has long been a favorite topic, but also for the mathematics class.

However, if magic squares are to be utilized to their greatest advantage, two simple but fundamental principles must be followed. I stumbled across these ideas about a year ago and have developed them in actual classroom teaching. The ideas are so simple that others must surely have come across them in their study of magic squares. However, I have not seen them set forth anywhere. I would be interested in knowing if they are embodied in any text or article.

If the first three numbers to be used in a 3×3 magic square are selected in accordance with the following two principles, the magic square will "check all the way around." Furthermore, the magic square may be multiplicative or additive.

PRINCIPLE I: In the case of the additive magic square, the three numbers first chosen must constitute a row, column, or a diagonal. The number which occupies the middle cell of the square must be the arithmetic mean of these first three, or, if the first three are placed on a row, column, or diagonal containing the center cell, the center number must then also be the arithmetic mean of the other two.

PRINCIPLE II. In the case of the multiplicative magic square, the three numbers first chosen must constitute a row, column, or a diagonal. The number which occupies the middle cell of the square must be the geometric mean of the first three, or the mean proportional to the other two if the first three are placed on a row, column, or diagonal containing the center cell.

If the first three numbers are placed in this latter diagonal position the fourth number may be chosen at random and entered at random. The remaining cells may then be filled in accordance with the operation to be employed.

If the first three numbers were placed in a row or column containing the center cell then the fourth number may be chosen at random but should be placed in a corner cell. This latter restriction is not mathematically essential, but ignoring it requires the determination of a fifth number which is undesirable for teaching purposes in junior or senior high school.

To prove these principles, enter a, b, c and d in four of the cells placing the first three in a row, column, or a diagonal. For the purpose of illustration, a diagonal has been used in Figure 1.

a		
	b	
d		c

FIG. 1. Magic Square at the Start.

If our magic square is to be additive, the

[1] THE MATHEMATICS TEACHER, vol. 44 (Jan. 1951), p. 43.

sum in any direction must be $(a+b+c)$, while if it is to be multiplicative, the product in any direction must be (abc). Using these properties, fill in three additional cells as shown in Figures 2 and 3.

a		$(a+c-d)$
$(b+c-d)$	b	$(b+d-c)$
d		c

FIG. 2. Additive Magic Square.

a		$\frac{ac}{d}$
$\frac{bc}{d}$	b	$\frac{bd}{c}$
d		c

FIG. 3. Multiplicative Magic Square.

If the middle row of the additive magic square is now added, the sum will be found to be $3b$. Since this must be equal to $(a+b+c)$, it follows that b must be the average or the arithmetic mean between a and c.

If the middle row of the multiplicative magic square is now multiplied, the resulting product will be found to be b^3. Since this must be equal to (abc), then b must be the geometric mean of a and c, or the mean proportional between them.

Note that the fourth number drops out in the process of obtaining the final relationship. Consequently, the choice of this number is arbitrary in this case. Further note should be taken of the similarity in the three additional cells in both types of magic squares provided they are entered by using the same directions.

The two sheets containing magic squares indicate the variety of purposes that such a device may serve.[2] Their history and their property of being self-checking all the way around furnish strong motivation in teaching. Additive magic squares enable a group to cover extensively the principles involved in the operations of addition and subtraction. The latter operation is involved in the process of obtaining any new number in the set. Similarly, multiplication and division may be given valued development through multiplicative magic squares.

Additive squares may include integers, fractions, decimals, percents, and denominate units.[3] Negative numbers have their place in the exercises. The need for subtracting in the obtaining of the desired sum has proved to be a worthwhile means of introducing the topic of signed numbers. Algebraic processes may be utilized by representing the unknown number as "x" and securing the equation needed to obtain the number.

Similarly multiplicative squares may be used for the presentation and development of the principles of algebra and arithmetic concerned with the inverse operations of multiplication and division. Work with square roots and irrationals may be treated through this type of square. The replacing of literal numbers by numerical values affords valuable practice in evaluation.

[2] Space limitations prevent the printing of the mimeographed sheets used by Dr. Rich in his classes. Interested readers may write him for copies.

[3] These are variations on the classical magic squares which contained only positive integers. Classically the subject has also been expanded to higher order squares, also magic cubes and circles. See for example W. W. Rouse Ball, *Mathematical Recreations and Essays*. (Macmillan and Co., 1940) pp. 193–221.

3	8	7
10	6	2
5	4	9

FIG. 4. Original Magic Square.

$3x$	$8x$	$7x$
$10x$	$6x$	$2x$
$5x$	$4x$	$9x$

$3x^2$	$8x^2$	$7x^2$
$10x^2$	$6x^2$	$2x^2$
$5x^2$	$4x^2$	$9x^2$

FIG. 5. These new squares may then be evaluated for varied values of x.

.3	.8	.7
1.0	.6	.2
.5	.4	.9

3%	8%	7%
10%	6%	2%
5%	4%	9%

3/3	8/3	7/3
10/3	6/3	2/3
5/3	4/3	9/3

FIG. 6. Arithmetic type magic squares obtained from the first one. The last of this group should be changed by the student into one consisting of mixed numbers.

3 ft.	8 ft.	7 ft.
10 ft.	6 ft.	2 ft.
5 ft.	4 ft.	9 ft.

which may be changed into

1 yd.	2 yd. 2 ft.	2 yd. 1 ft.
3 yd. 1 ft.	2 yd.	2 ft.
1 yd. 2 ft.	1 yd. 1 ft.	3 yd.

FIG. 7

Creative formulation of magic squares may be carried on by students as long as the two fundamental principles are observed. Self-reliance is strengthened through the process of making up new magic squares, completing the ones made up, and then checking them "all the way around." Note how many new magic squares may be invented from one that has already been finished:

Finally, the two types of magic squares can be interrelated by using the values of the additive magic square as the exponents in a multiplicative magic square as shown in Figures 8 and 9.

3	8	7
10	6	2
5	4	9

FIG. 8. Additive Magic Square.

x^3	x^8	x^7
x^{10}	x^6	x^2
x^5	x^4	x^9

FIG. 9. Multiplicative Magic Square.

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37. Still More About Nedians

The following letter to the editor is a good preface to some additional notes on nedians as well as being of interest in itself:

1401 East 35th Street
BROOKLYN 34, N. Y.
May 29, 1951

DEAR SIR:

I am a student at James Madison High School in Brooklyn. While glancing through the January 1951 issue of THE MATHEMATICS TEACHER, my attention was attracted to the article entitled *Nedians of a Plane Triangle.* Upon inquiring, I was told that several other articles had appeared in your more recent issues.[4]

This subject was of particular interest to me because I had prepared a thesis on *inwoven polygons* for the Westinghouse Science Contest of December, 1950.

In my thesis, using plane geometry, I not only developed the solutions which your magazine printed, but went further to establish a general formula for all inwoven regular polygons. I ended by proving by analytical geometry that the parallelogram formula does not apply to irregular quadrilaterals.

[4] Notes on nedians and associated topics are to be found in *Miscellanea 18, 26, 32, 34,* in THE MATHEMATICS TEACHER, Vol. 44 (January 1951), pp. 46–48; (May 1951), pp. 310–312; (Nov. 1951), pp. 496–500.

Mr. George Morgenstern, the faculty adviser of the James Madison High School "Math Team," has urged me to send you a copy of my thesis which I am enclosing.

Respectfully yours,
MARILYN R. TAIG

We regret that we cannot print all of Miss Taig's thesis, but we do reproduce her last theorem because it provides an interesting addition to those which we have printed so far and also displays her method of procedure.

THEOREM. If in any parallelogram, $RSTU$, n segments of a units length and n segments of b units length are laid off alternately from each vertex in a cyclic order, and if each point of division is connected to the opposite vertex, then the area of the inwoven parallelogram, C, is

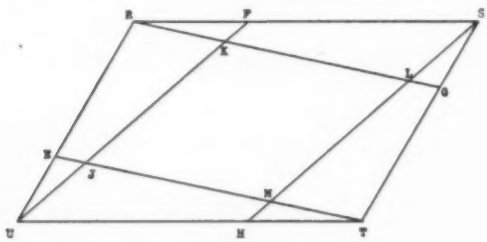

FIG. 10

given by $C=(n-1)^2/(n^2+1)\cdot K$, where K is the area of the parallelogram.

In Figure 10:

$$\triangle STH=\triangle TUE=\triangle URF$$

(1) $$=\triangle RSG=\frac{K}{2n}$$

(2) $\triangle SGL=\triangle UEJ;\quad \triangle THM=\triangle RFK$

(3) $MLGT=JMHU=KJER=KFSL.$

Let B represent the area of these latter quadrilaterals, then

(4) $$\frac{\triangle UEJ}{\triangle UEJ+B}=\frac{a^2}{a^2n^2}=\frac{1}{n^2};$$
$$\frac{\triangle RFK}{\triangle RFK+B}=\frac{b^2}{b^2n^2}=\frac{1}{n^2}.$$

(5) Hence $\triangle UEJ=\triangle RFK$.

Let A represent their area.

Then $\frac{A}{A+B}=\frac{1}{n^2}$, and

(6) $$B=A(n^2-1).$$

Substituting from (6) in $C+4B+4A=K$ we have

(7) $$K=C+4An^2.$$

But $2A+B=K/2n$ from (1) and (5) which, by substituting from (6), becomes

(8) $$A(n^2+1)=\frac{K}{2n}.$$

Eliminating A from (7) and (8) finally gives

$$C=\frac{(n-1)^2}{n^2+1}\cdot K.$$

Miss Taig's attack on the trapezoid via a coordinate system and the use of determinants to express both collinearity and area lead to the conclusion which she noted in her letter in regard to the general quadrilateral. Perhaps someone else has a neat formula for the *inwoven*, *cevian*, or *redian* figure of this polygon?

38. Monkeys and Coconuts

THE PROBLEM: A group of persons have a quantity of coconuts to be divided equally among themselves. Each person in turn, not trusting his friends, surreptitiously, sorts the coconuts into piles, buries his own share and jumbles the remainder together. However, in order to have all piles equal each person had to give the same number of coconuts to some waiting monkeys. How many coconuts were there at the start?

Most forms of the "monkeys and coconuts" problem not only can be appreciated but can be solved by the alert student of high school mathematics. In support of that opinion some solutions are offered.

NOTATION: $A, B, C, D, E, \ldots$ in succession are the numbers of nuts in a pile, A at the start, B after the first operation, and so on.

EXAMPLE 1. Three men, one monkey, four "operations." In this case *operation* means to give one coconut to the monkey and then to carry away one-third of the remaining pile.[5]

[5] This problem is to be found in Maurice Kraitchik, *Mathematical Recreations*. (New York: W. W. Norton and Co., Inc., 1944), p. 32.

The problem is to find positive integers, A, B, C, D, E, to satisfy

$$\begin{aligned} 2(A-1)&=3B,\\ 2(B-1)&=3C,\\ 2(C-1)&=3D,\\ 2(D-1)&=3E. \end{aligned} \tag{1}$$

From equations (1) the student can eliminate B, C, and D and arrive at

$$16A=81E+130. \tag{2}$$

If he cannot solve this by trial-and-success, the problem must be abandoned at this point. It will be of little use to tell him about congruences, or continued fractions, or "indeterminate" equations.

Notice now that equations (1) can be written in the form:

$$\begin{aligned} 2(A+2)&=3(B+2),\\ 2(B+2)&=3(C+2),\\ 2(C+2)&=3(D+2),\\ 2(D+2)&=3(E+2). \end{aligned} \tag{3}$$

This amounts to importing two blue coconuts. They will make the arithmetic easier for the men and make life harder for the monkey. They are given a distinctive mark because they are to be exported at the end. They may be regarded as catalysts; they make the game easier without really playing in it "for keeps."

From equations (3) by multiplication,

$$16(A+2)=81(E+2). \tag{4}$$

Naturally this is the same as (2) but it is much more convenient.

Since E is a positive integer, so also is $(E+2)$. It follows that $16(A+2)$ is divisible by 81. Now 16 and 81 have no useful common factor and therefore $(A+2)$ is divisible by 81. By similar reasoning, $(E+2)$ is divisible by 16.

Note that (4) can now be written in the form

$$\frac{A+2}{81}=\frac{E+2}{16}=K$$

where K is any positive integer.

The chestnut is cracked. By going back to equations (3) it is easy to show

$$\frac{A+2}{81}=\frac{B+2}{54}=\frac{C+2}{36}=\frac{D+2}{24}=\frac{E+2}{16}=K$$

where $K=1, 2, 3, 4, \ldots$.

We have thus the "one-parameter" set of solutions:

$$\begin{aligned} A&=81K-2,\\ B&=54K-2,\\ C&=36K-2,\\ D&=24K-2,\\ E&=16K-2, \end{aligned}$$

where K is any positive integer.

Quod erat inveniendum.

INTERMISSION: It is recommended that the student take time to tabulate a few successive solutions.

K	1	2	3	4	5
A	79	160	241	322	403
B	52	106	160	214	268
C	34	70	106	142	178
D	22	46	70	94	118
E	14	30	46	62	78

He should find it easy to discover what law governs the numbers in any row and what law governs the values of $(A-B)$, $(B-C)$, $(C-D)$, $(D-E)$, taken from any column.

EXAMPLE 2. Three men and 403 monkeys arrive at twilight on a tropical island where there is a pile of coconuts too big to be counted before dark. It is assumed that the men know enough mathematics to be able to "deal," and that the monkeys are sufficiently polite to refrain from snatch and grab. One man, while the others sleep, gives a coconut to each monkey, deals the rest into three equal piles, takes away one pile for himself, puts the other two piles neatly together and goes back to sleep. In succession the second man and the third do likewise. How many nuts were in the original pile?

For finding A, B, C, D, we have

$$2(A-403)=3B,$$
$$(6)\qquad 2(B-403)=3C,$$
$$2(C-403)=3D.$$

Import 806 blue coconuts observing that

806 = (number of sleeping men) · (number of monkeys).

Equations (6) in handier form lead to (7) and (8)

$$2(A+806)=3(B+806),$$
$$(7)\qquad 2(B+806)=3(C+806),$$
$$2(C+806)=3(D+806).$$

$$(8)\qquad 8(A+806)=27(D+806).$$

By the sort of reasoning used before, $(D+806)$ must be divisible by 8. We have as possible values of D: 2, 10, 18, 26, . . . That is $D=2+8t$, where $t=0, 1, 2, 3, 4, \ldots$ Put this value of D into equation (8).

$$8(A+806)=27(2+8t+806)$$
$$=27(808+8t)$$
$$A+806=27(101+t)$$
$$A=1921+27t.$$

The pile contains at least 1921 coconuts but may contain 1948, 1975, 2002, . . .

What we wanted.

NOTE: There is no magic about choosing 806 instead of any other auxiliary number. If we set

$$2(A-403)-3B=2(A+x)-3(B+x),$$

we shall find $x=806$.

EXERCISE 3. n humans, 1 monkey, n "operations." This problem is used to introduce an interesting article on the theory of numbers in the *Scientific American*.[6] The alert student can now solve this problem. The student who never wants to see another coconut problem may still want to verify the answer:

$$A+n-1=K(n^n).$$

The number K is to be a positive integer and it can be chosen large enough to make the last population of the pile a *positive* integer. After that has been attended to, any integer larger than that value of K will serve equally well.

For the teacher and the insatiable student there remains

EXERCISE 4. n humans, s simians, $(n+1)$ operations.

FOOTNOTE: The trick of "importing" is not novel. For instance you may recall the whimsy about Omar ben Solomon, the sheik who was called in to distribute an estate of 17 camels giving one-half to the eldest son of the testator, one-third to the second son, and one-ninth to the remaining son. He did it without bloodshed and enhanced his reputation for wisdom by putting his own camel in the corral with the others. After the sons had been given 9, 6, and 2 camels, he embarked on his own and sailed away.[7]

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University of Michigan
Ann Arbor, Michigan

[6] Paul S. Herwitz, "The Theory of Numbers." *Scientific American*, Vol. 185, No. 1 (July, 1951), pp. 52–55. Since this note was written, *The Scientific American* for September, 1951 has appeared with letters about the article and a *general* solution by the original author on pages 2, 4, 6. P.S.J.

[7] *Department Editor's note:* Don't miss the difference however. Omar ben Solomon's "wisdom" not only avoided "halving" a live camel but also politely ignored the bad arithmetic of the deceased. Professor Anning's "imported" coconuts facilitate good mathematics.

HAVE YOU SEEN?

"Engineers" by Karl T. Compton in *Scientific American*, September, 1951. "There are 400,000 engineers, but they are in acutely short supply. An analysis of this critical situation, with some suggestions as to how it might be alleviated in the future."

"Scientists" by M. H. Trytten in *Scientific American*, September, 1951. "We have 175,000 and need more. The problem is complicated by the fact that applied science takes workers away from pure science, the wellspring of our technological progress."

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. Or if that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

TINKERTOY MODELS[1]

Editor's Note: An important aspect of the "building" procedure outlined in this article is its developmental nature. The technique is unique; the contributor illustrates with a solid geometry proposition how the building of a model in itself becomes part of the proof. The reader will note as he proceeds with this article that the "Given" is built first. E.J.B.

As an illustration of the use of TINKERTOY in the solid geometry classroom consider the proof of the following proposition: "If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to their plane."

Begin by compressing two rods, BC and BF, into the rim holes of a spool. Let these two rods represent the given intersecting lines. (See Fig. 1.) Next compress a third rod AB into the through hole of the spool; AB will be perpendicular to lines BC and BF. Thus the "Given" part of the proof is now completely illustrated.

By compressing a fourth rod BE into a rim hole it is possible to represent "any line through the given point lying in the plane of the given intersecting lines." Extend line AB through B to A′ by compressing a fifth rod into the through hole of the spool on the side which is opposite A. Rod A′B should be selected so that AB = A′B. At this point the student should be asked to note that end A of rod AB represents a point on a line of unlimited length and that AB represents only a segment of that line.

The model may be completed by using colored pieces of ribbon with knots tied on their ends. The pieces of ribbon slip readily into the slots at the ends of the rods and are held in place by the knots. The ribbons may be thought of as representing the construction lines in the figure. They are helpful in locating the various triangles which must be considered in stating the proof.

One advantage of the model building idea suggested in this article is that students can see how the figure is related to the statement of the proposition. Only the materials are brought to class—not the completed model. The given facts are shown first. Then the construction lines are added. After this the proof usually follows with comparative readiness.

The building procedure described above will be found helpful in nearly all work with lines and planes and to a limited extent with prisms.

HELEN MACDONALD SIMMONS
Governor Dummer Academy
South Byfield, Massachusetts

RIGHT TRIANGLE MODELS

Illustrated in this article is a set of models which can be used to help develop the statements and proofs of those right triangle theorems which have to do with the altitude upon the hypotenuse. The set

[1] Directions for construction in this article are given in terms of the language employed in the "Direction Sheet for Wonder Tinkertoy," The Toy Tinkers Inc., Evanston, Illinois, 1950. TINKERTOY is available from the manufacturers at this address.

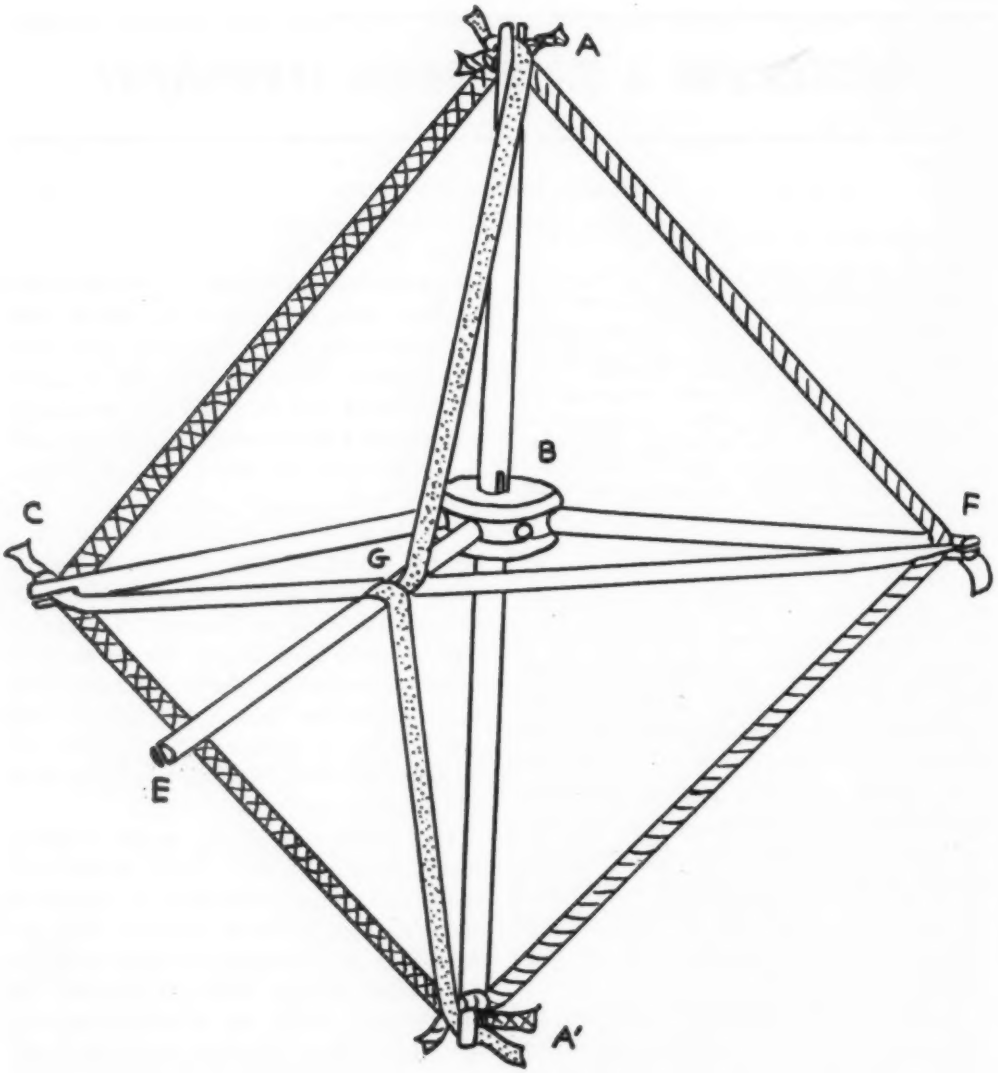

FIG. 1

consists of seven right triangles which may be made from plywood and painted different colors as illustrated by the shading in the diagrams, or cut from heavy colored cardboard.

For the sake of convenient reference to

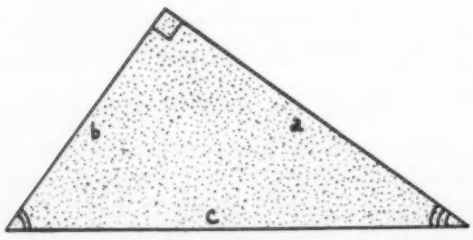

FIG. 2

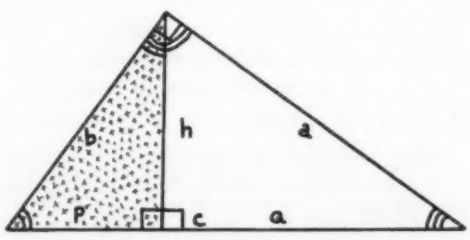

FIG. 3

the drawings let us refer to the largest of the right triangles as the "original" one. Suggested dimensions for its hypotenuse and altitude are 18″ and 8″ respectively. Three of these and two each of the smaller right triangles should be constructed.

Figure 2 represents the original right triangle; Figure 3, the original triangle with the altitude upon the hypotenuse as the common side between the other two tri-

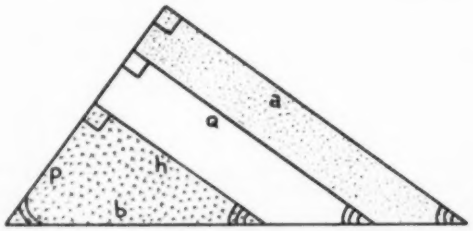

FIG. 4

angles, and Figure 4 shows the three right triangles separated. Please note that there are seven triangles in the three illustrations.

By matching the three triangles illustrated in Figure 4 with their duplicates it is possible to show that the triangles of Figure 2 and 3 are all similar.

By employing the set of models described above in appropriate laboratory experiments the following geometric facts may be developed and demonstrated:

(1) The altitude upon the hypotenuse of a right triangle divides the triangle into two triangles which are similar to each other and to the original triangle.
(2) If the altitude to the hypotenuse of a right triangle is drawn:
    (a) The altitude is the mean proportional between the segments of the hypotenuse;
    (b) Either leg is the mean proportional between the hypotenuse and the adjacent segment.
(3) The product of the two legs is equal to the product of the hypotenuse and altitude (twice the area of the original triangle).
(4) The mean proportional construction.
(5) The Pythagorean Theorem.

ISADORE CHERTOFF
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Bayonne, New Jersey

BILLIARD MARKERS FOR ABACUSES

Mr. Frank Hawthorne of Hofstra College has informed us that wooden billiard markers may be procured in quantity from The Brunswick-Balke-Collender Company, 109 Fifth Avenue, New York 3, N. Y. These beads are very good ones for making abacuses and fact finders. The current retail price for a set of 100 markers complete with wire hooks and stretchers is $3.30 regardless of quantity. The 100 markers may also be purchased separately at a cost of $2.30.

A SIMPLE STADIA TUBE

Measuring distances with a stadia tube is an interesting application of similar triangles. The instrument as usually described in books on field work consists of a tube with a peep slight at one end and a pair of parallel cross wires at the other. Depending on the materials used in construction it is possible to produce some very precise instruments.

The device suggested here can be pro-

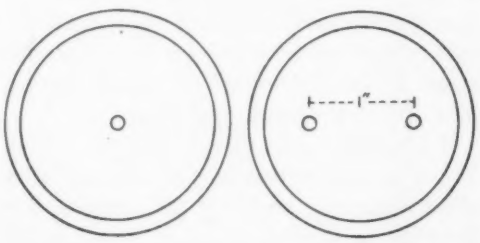

FIG. 5

duced with two two-piece Kerr Mason fruit jar caps, a piece of tagboard 12″ square, and a small roll of cellophane tape.

Disassemble the two jar caps—that is, remove the lids from the screw bands. Drill a $\frac{1}{8}$″ hole in the center of one of the lids, and drill two $\frac{1}{8}$″ holes 1″ apart (from

center to center) in the other lid. (See Fig. 5.) The metal in the lids is so soft that the holes can also be made with the metal point of compasses or a paper punch.

To assemble the tube replace the lids in the screw bands, roll the 12″ square of tagboard into a tube and fit the assembled jar caps around the ends of the tagboard tube. (See Fig. 6.) A little cellophane tape will secure the jar caps to the tagboard.

For most practical purposes this device is rather crude, but it has the advantage of simplicity and the materials needed to produce it cost very little. Every student in class can produce one in a few minutes. This fact in itself would seem to compensate for its lack of fineness. It is not exactly a secret among mathematics teachers that lessons involving field work are difficult to conduct because of a lack of equipment. Usually there are too many spectators and not enough participants.

To measure any distance such as D in Fig. 6 procure a broom stick, fish pole, or other long rod, and two red ribbons. Tie the ribbons around the rod as shown in the figure. If one student holds the rod at eye level in a position which is horizontal with the ground and parallel to the end of the tube through which another student is sighting, the distance D may be readily computed. The student doing the sighting must line up the ribbons through the two holes in the far end of the tube. It is his responsibility to direct the student holding the rod to slide the ribbons together (or apart) until the line-up is effected.

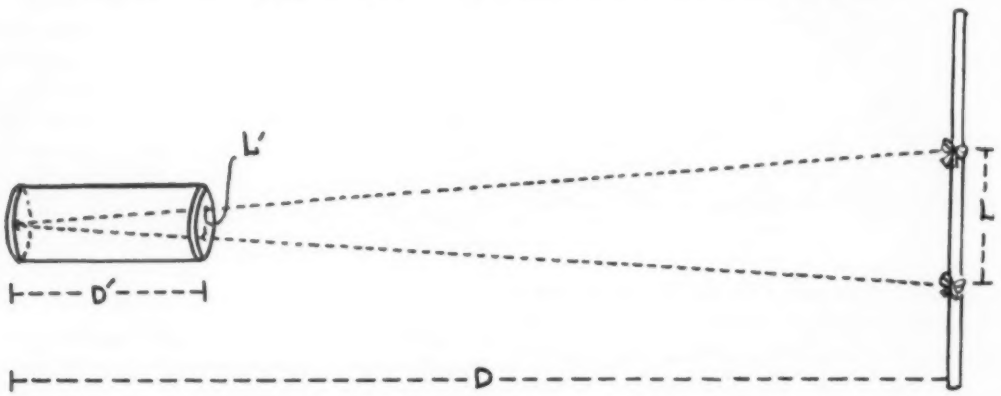

FIG. 6

After measuring the length L, the distance D may be obtained directly from the following proportion by making the appropriate substitutions for L, L', and D':

$$\frac{D}{D'}=\frac{L}{L'}.$$

Note that $L'=1''$, and $D'=12''$. The proportion is a direct application of the fact that the altitudes of two similar triangles are to each other as their bases.

THE MATHEMATICS LABORATORY
Monroe High School
St. Paul, Minnesota

HAVE YOU SEEN?

"Geometry in Flags" by Alice Lamb, and "Geometric Designs in Life" by Octavia Beard in *Student Life*, November, 1951.

"Classroom Atmosphere Stimulates Arithmetic Study" by Edna M. Nolan in *Educational Progress in the Chicago Public Schools*, October, 1951.

"Swimming is a Science" by Georg Mann in *Science Digest*, July, 1951. "The square of the maximum speed of the total stroke is the sum of the squares of the separate swimming speeds of the arms and legs."

"Beware of Bumper-Chasers" in *Science Digest*, July, 1951. "A rule of thumb the experts use for stopping on dry pavement is to square the speed you're driving, and divide it by 10."

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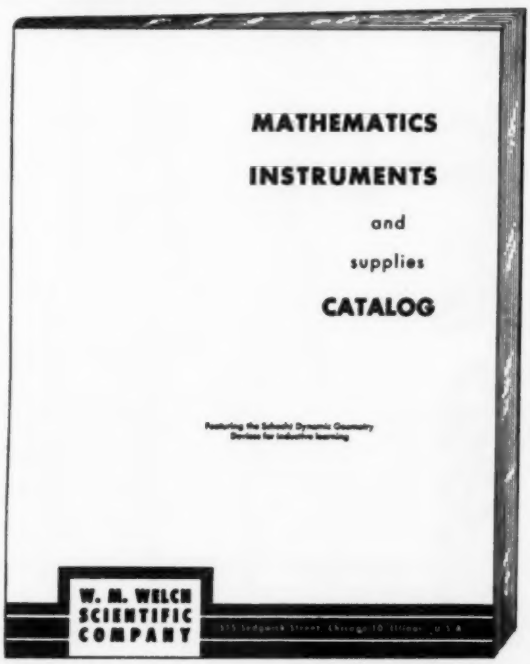

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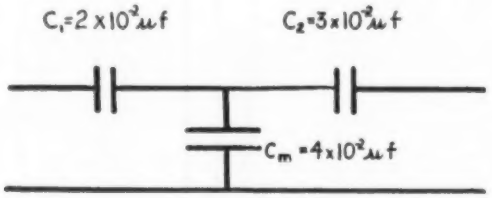
THE MATH:

The Algebraic Form

$$X=\frac{\sqrt{(a\,b)}}{\sqrt{(c+a)}\ \sqrt{(c+b)}}$$

The operations appearing in this radio application:

1. $\sqrt{(ac)(bc)} = c\sqrt{(a\,b)}$
2. $\sqrt{(ac^2+bc^2)} = c\sqrt{(a+b)}$
3. $\sqrt{(a^{-2m})} = a^{-m}$
4. $\frac{a^{2m}}{a^m\,a^m} = 1$



C_1, C_2 and C_m are capacities of the condensers, and μf stands for microfarad (one millionth of a farad, the unit of capacity).

THE RADIO:

Coefficient of Coupling(k)

$$k=\frac{\sqrt{(c_1\,c_2)}}{\sqrt{(c_m+c_1)}\sqrt{(c_m+c_2)}}$$

$$=\frac{\sqrt{(2\times10^{-2})(3\times10^{-2})}}{\sqrt{(4\times10^{-2})+(2\times10^{-2})}\ \sqrt{(4\times10^{-2})+(3\times10^{-2})}}$$

$$=\frac{10^{-2}\sqrt{(6)}}{10^{-2}\sqrt{(8)}\ 10^{-2}\sqrt{(7)}}=\frac{1}{\sqrt{(7)}}$$

$$=.38-$$

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POSTER # 13

EXPONENTS and RADICALS enter into STUDY of RESONANCE

MATH IN RADIO

When you tune a radio receiver to the signal from a broadcast station, you adjust the input circuit of the receiver to resonate at the frequency of that station.

THE ELECTRICAL PROBLEM:

To what frequency is this circuit resonant?

find f

$L = 25\ \mu h$

or $L = 25 \times 10^{-6}$ henries

THE FORMULA $f = \frac{1}{2\pi\sqrt{LC}}$ when

f = frequency in cycles

L = Inductance in henries

C = Capacitance in farads.

$\pi = 3.14$

$$f = \frac{1}{6.28\sqrt{25 \times 10^{-6} \times 100 \times 10^{-12}}}$$

$$= \frac{1}{6.28\sqrt{25 \times 10^{-16}}}$$

$$= \frac{1}{6.28 \times 5 \times 10^{-8}}$$

$$= \frac{10^8}{31.4} = 3.18 \times 10^6 \text{ cycles}$$

or 3.18 megacycles.

PREPARED IN COLLABORATION WITH THE MEN'S AND WOMEN'S MATHEMATICS CLUBS OF CHICAGO

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POSTER # 14

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APPLICATIONS

Edited by SHELDON S. MYERS
Department of Education, Ohio State University, Columbus, Ohio

WE HAVE received a number of request for military applications of high school mathematics. There are unlimited possibilities in this direction, and this department can only present a few examples suggestive of the larger field. We are using the examples in this issue to provide references to some sources of applications of a military nature.

P.G. 5 Gr. 10–12 *Plane Geometry and Artillery Firing*

The following application is number 5 in the pamphlet, "Some Military Applications of Elementary Mathematics," by the Department of Mathematics, the United States Military Academy, June 1942, prepared for The Institute of Military Studies, The University of Chicago.

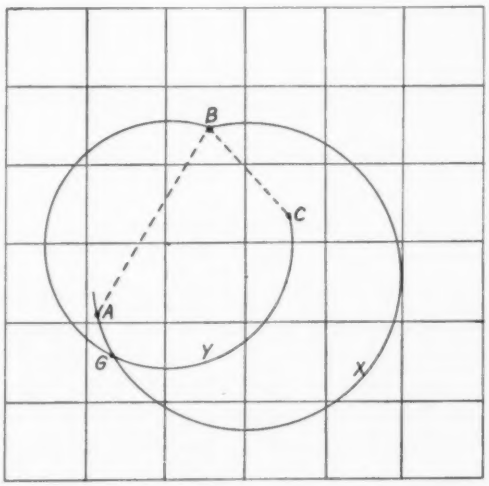
This problem is often encountered in military surveying. In the field, the problem is usually solved by another method, depending on certain items of surveying equipment. However, the solution given below, based on compasses and straightedge, illustrates the basic principle of solving.

"*Mathematical Features*: Construction of a segment which shall contain a given angle; intersection of loci.

"*Problem*: During an artillery reconnaissance, a gun position was selected, and the reconnaissance officer was instructed to plot its location accurately on the map. From the gun position, he was able to see three prominent landmarks, A, B, and C, which had previously been accurately located and plotted on the map. (See accompanying grid sheet.) Setting up an instrument at the gun position, he read angles as follows:

A-Gun-B = 800 mils or 45°
B-Gun-C = 533 mils or 30°

"*Requirement*: from the foregoing data, determine the map location of the gun position.

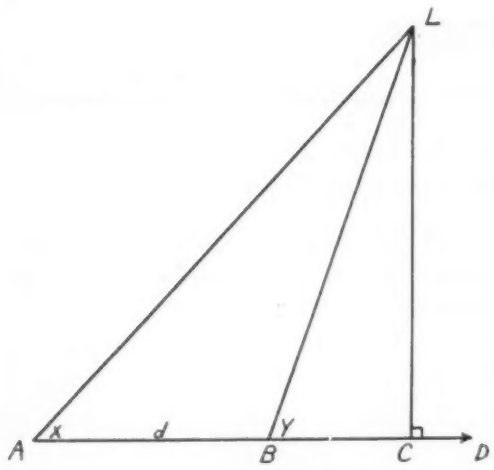


"*Solution*: Upon AB, construct a segment (AXB) which shall contain inscribed angles of 800 mils or 45°. Upon BC, construct a segment (BYC) which shall contain inscribed angles of 533 mils or 30°. The intersection of arc AXB with arc BYC is the gun position, G."

T. 3 Gr. 10–12 *How Far Is the Lighthouse?*

Problems of navigation usually play an important part in war time mathematics. Here is an application involving simple trigonometry and algebraic manipulation from "Wartime Applications of Mathematics," by John J. Kinsella, University School Series No. 1, Ohio State University, 1945, 31 pp. From page 23:

"A ship sailing in the direction AD observes that angle $LAD = x°$. After a run of d miles to B it notes that angle $LBD = y°$. What will be its distance from lighthouse L when it is "abeam" at point



C, so that angle $LCD = 90°$? Show that:

$$CL = \frac{d}{\cot x - \cot y}."$$

Ar. 12 Gr. 6–10 *Changing Metric Information about a Famous German Warplane to English Units*

The War Service Program of the Civil Aeronautics Administration was one aspect of a stepped-up program of youth education for the air age precipitated by the last war. Complementing this program was the publication of a number of textbooks in basic mathematics for aeronautics. One of these was *Basic Mathematics for Pilots and Flight Crews* by Newsom and Larsen, Prentice-Hall, 1943, 153 pp. The following is from page 73.[1]

"The following information is listed in regard to the famous plane, the Ju 49, developed in Germany in 1928.

Span: 28.26 m.
Length: 17.20 m.
Height: 4.75 m.
Weight: Loaded, 4,250 kg.
Maximum speed: 146 km./hr.
Greatest height reached: 12,739 m.

"Rewrite this information using the English system of units. Students would probably be eager to look up whatever metric data is available on the new Russian jet planes and convert it to English units.

[1] Reprinted by permission of Prentice-Hall, Inc. and the University of New Mexico Press.

Al. 10 Gr. 9–10 *Ratios of Planes at a Training Base*

Another problem is taken from page 41[1] of the above book.

"At a bombardier training base, there were ten AT-11's for each B-24 stationed at the base; but there were two of the B-24's for each B-17. How many planes of each variety were located at the base if there were 138 planes in all?"

C. Al. 5 Gr. 10–12 *Finding the Temperature of an Explosion*

The book *Engineering Problems Illustrating Mathematics* by John W. Cell, McGraw-Hill Book Company, 1943, 172 pp., contains an extensive collection of valid engineering problems arranged in five parts covering college algebra, trigonometry, analytic geometry, differential calculus, and integral calculus. Each of these parts are subdivided into subtopics. There are great numbers of problems which can be used on the high school level such as the following from page 4:[2]

"The temperature t of explosion (in degrees centigrade) of nitroglycerin is given by a solution of the equation:

$$(a+bt)t = 1000Q_v + 15(a+15b)$$

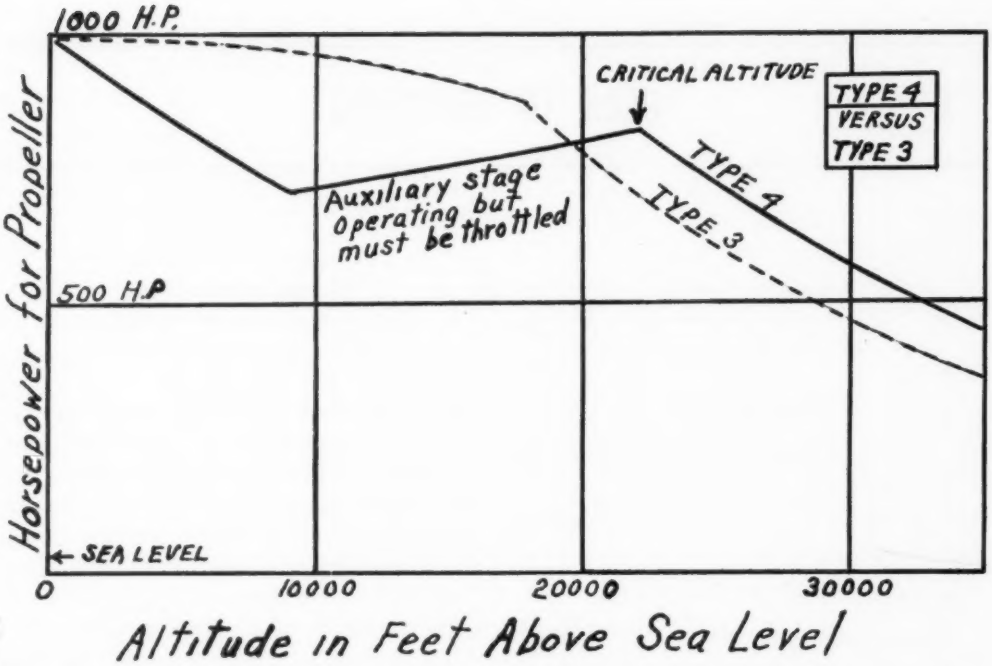
where, for nitroglycerin, $a = 43.705$; $b = 0.01775$; $Q_v = 346.5$. Determine the temperature t. Determine the temperature of explosion for ammonium picrate where the data for the formula are as follows: $a = 54.04$; $b = 0.01212$; $Q_v = 175.3$."

These problems come under "Quadratic Equations" in the "College Algebra" part.

Al. 11 Gr. 9–10 *Performance Graphs of Superchargers*

The ability to interpret graphs is one of the most important outcomes of secondary mathematics. Why confine the study of graphs in ninth and tenth grade algebra to the abstract variables X and Y? The study and visualization of functions through graphs of interesting concrete var-

[2] Reprinted by permission of the Editorial Committee of the Mathematics Division of the Society for the Promotion of Engineering Education, John W. Cell, Chairman, and the McGraw-Hill Book Company.



iables is mathematically fruitful and psychologically motivating. In this connection a source which has proved to be very worthwhile is the 80 page pamphlet entitled "Airplane Power, with Special Reference to Engines and Altitudes" published by General Motors in 1943. Boys in particular have been challenged by this clearly written manual on seven types of airplane engine superchargers and their altitude performance graphs. A teacher without any background could easily read the manual in one or two hours and understand its essential features. The manuals were free and could be ordered in quantity by writing Room 11–230, General Motors Corporation, Detroit, Michigan.

Superchargers are special blowers attached to the fuel intake systems of aircraft engines to offset the effects of low oxygen due to high altitudes. Seven different, graduated types are clearly and simply diagramed and explained. The performance of each type is compared graphically with the preceding type of supercharger. The graph above,[3] taken from the manual, compares Type 4 with Type 3. Type 4 is a two-stage supercharger with mechanical clutch, while Type 3 is a single stage supercharger with variable speed clutch. Type 4 consists of two blowers, one running continuously, while the other one is put into operation at 9000 feet. Notice what happens to the graph for Type 4 at 9000 feet. Students can be asked to compare the two types, giving the advantages and disadvantages of each as shown by the graph.

Some "Tail End" Data

Research has shown that a cocker spaniel wags his tail with a frequency of about 300 vibrations, or wags, per minute. More profound investigation of the problem revealed that the end of a four-inch tail traveled 3 inches with each wag, giving a tip speed of 900 inches per minute or 85 hundredths of a mile an hour.

[3] By permission of General Motors Corporation.

"How Children Use Arithmetic" by Effie G. Bathurst is a new Office of Education Publication which shows how children develop arithmetic abilities. It is listed as Bulletin 1951, No. 7, contains 13 pages and may be purchased from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. for 15 cents. Catalog No. FS 5.3:951/7.

AIDS TO TEACHING

Edited by

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and

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BOOKLETS

B. 83—Use, Care, and Adjustment of Polar Planimeters

Keuffel & Esser Co., 127 Fulton Street, New York, N. Y.

Pamphlet; 9″×6″; 14 pages; $.50

Description: After a brief description of the planimeter and its parts, the operation of ascertaining an area is presented in detail. This operation consists in the proper placement of the device, the movement of the tracer on the perimeter of the area to be measured and the reading of the measuring wheel. Variations of these instructions are given for different types of planimeters. Suggestions are included on the care of the instrument so that maximum accuracy and use will be attained from each instrument.

Appraisal: This pamphlet will give the teacher a simple explanation of how the planimeter works as well as how to use it to measure area. Probably the reason planimeters have so seldom been used by mathematics teachers as an example of a measuring device or as a check of area computation has been a lack of knowledge of how they work. This pamphlet will be useful in showing teachers another device to be used for measurement activities.

B. 84—Ranger 'Rithmetic

U. S. Department of Agriculture, Forest Service, Washington, D. C.

Booklet; 8″×10″; 12 pages; Free

Description: The main part of this booklet is concerned with 21 problems in arithmetic. It is stated on the cover that these are intended for seventh grade classes, but the numbers and vocabulary may not be suitable for some groups at that level. Each problem is accompanied by a black and white drawing, and has the answer printed under it in smaller type. Then there follows a message about conservation which pertains to the problem. Here is one of the shorter ones: "Problem No. 6. There are 4,200,000 small forest ownerships in the United States and 97% of them are east of the Great Plains. How many are east and how many are west of the Great Plains? Answer: 4,074,000 east; 126,000 west. Small forests are the greatest part of our forest problem because there are so many of them and because the owners are often indifferent toward good care of them." There is a list of materials to help teach forest conservation.

Appraisal: The problems are often as artificial as many of the common problems in arithmetic books, but the cumulative effect of so many on one subject, and the reality of the numbers makes this booklet a good supplement in teaching. The little pictures add a great deal to the interest. There is no reason why conservation should not be taught every time that it is possible to do so. It is difficult to assay the actual value of forest saved by the publication of this booklet, but then much of the expenditure of the government is removed from its ultimate intended effect. There is no doubt that the objective is worth while (but what equally worthy projects were

excluded by this one?). When materials such as this exist, teachers should certainly make use of them.

B. 85—Standard Time Throughout

Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

Booklet; 6″×9″; 30 pages; $.15

Description: This booklet, by R. E. Gould, is a publication of the National Bureau of Standards. It begins with a short historical sketch of standard time, and then contains two long sections on "Standard Time in the United States," and "Time in Foreign Countries." Some of the topics treated are time zones, time signals, international date line, time designations, and daylight saving time. There is a long table comparing the standard times in countries throughout the world. A bibliography is included.

Appraisal: It is not important to consider whether *all* the material in the booklet would be used in a mathematics classroom. So many of us are textbook minded that it seems wrong to use advanced material which is read only in selections. And yet, this process of selecting and discarding parts of a work is seldom taught in mathematics classes. The exercise of judgment is neglected. For this and other reasons there is a place for this interesting booklet. The use of plus and minus signs to signify zone numbers is practical and enlightening. The understanding of denominate numbers, hours—minutes—seconds, is important. This would be a very valuable addition to a classroom library of supplementary materials.

B. 86—How to Choose a Slide Rule

Keuffel and Esser Company, 127 Fulton Street, New York, N. Y.

Booklet; 5½″×8¼″; 22 pages; Free

Description: The booklet begins with a rather general discussion of advantages of performing operations on a slide rule. Then the advantages and disadvantages of specific types of rules are presented. The models included are the Mannheim Polyphase, Polyphase Duplex Trig (and Decitrig), Log Log Duplex Trig (and Decitrig), and the Log Log Duplex Vector.

The story-like account is very humorously written and interestingly presented. The booklet is printed in black and red upon white pages. It is excellently illustrated by fifteen cartoons and photographs of each rule which is discussed. The last five pages are devoted to advertising which is hardly objectionable because it is so informative about prices and availability of slide rules in general.

Appraisal: The booklet is excellent for teaching high school students to select a specific slide rule to fulfill predetermined desires. Although the discussion is intended primarily for college students, it is perfectly suitable for college preparatory students in the later years of high school; slide rules should become familiar instruments by the time a student enters college, and proper instruction on the high school level serves to acquaint the student with the model he needs. The necessary facts are presented in the booklet in a form simple enough for the student to understand.

The booklet is not suitable for use in the ninth and tenth grades because students have not yet encountered many of the problems which make the slide rule a beneficial instrument. Several copies of the booklet are sufficient to satisfy the needs of the entire class; they may be circulated among the pupils. (Reviewed by Bernard Singer, Hyannis, Mass.)

B. 87—Map Projection Studies, No. P208

Denoyer Geppert Company, 5235 Ravenswood Avenue, Chicago 40, Ill.

Booklet; 8½″×11″; 20 pages; $0.50

Description: Many types of map projections are compared by means of illustrations, questions to the reader and a short textual description. The following projections are discussed: orthographic,

stereographic, globular, gnomonic, polar equidistant, polar equal area, stereographic cylindrical, gnomonic cylindrical, mercator, homolographic, sinusoidal, semi-elliptical, homolosine, simple conic, conic two standard parallels, conformal conic, polyconic, Albers, and Bonne.

Appraisal: Naturally the meaning, definition and derivation of these projections is not intended at the secondary school level. However, it is very convincing to thumb through these pages and see how the facts of plane and solid geometry are put to use in geography. The illustrations are remarkably clear and helpful.

B. 88—Study Guide and Work Book (Life Insurance)

B. 89—Set of Questions (Life Insurance)

Institute of Life Insurance, 488 Madison Avenue, New York 22, N. Y.

Booklets; $8\frac{1}{2}'' \times 11''$; Free

Description of B. 88: This is a workbook (40 pages) to be put into the hands of the pupils in a class which has studied the "Handbook of Life Insurance" (B. 12—November, 1948). It is framed in terms of the meanings and descriptions of life insurance; very little inspiration for computation is included.

Appraisal of B. 88: This is an extremely complete booklet on the facts on life insurance—far too complete. Few schools would find it worth their while to spend the amount of time on the subject which this booklet would demand. It will be tempting to do so, to the detriment of other subjects, but if proper selection is made, this booklet should be a tremendous help.

Description of B. 89: On 8 pages there are 20 true-false, 20 multiple choice questions and 20 completion questions with answers and references for instructors to use.

Appraisal of B. 89: The questions are not too well written, but suffer from the common ambiguities of objective-type questions when poorly written: they often make little sense without their answers. The amount of detail expected is much too much for the time spent on insurance in today's schools. However, by selecting and adapting, the material will become useful to teachers.

B. 90—Elementary Instructions for Operating the Slide Rule

B. 91—Graph Sheets

B. 92—Templates and Other Drafting Tools

B. 93—Drafting and Reproduction Equipment and Materials

Keuffel and Esser Co., 127 Fulton Street, New York, N. Y.

Booklets; $5\frac{3}{4}'' \times 8\frac{3}{4}''$; Free

Description of B. 90: This 19-page booklet is useful only to those who have slide rules in their hands, and most useful to those with K & E rules. It tells how to read scales, how to multiply, divide, square, find square root, use proportion, the CI scales, cube and cube root scales, and trigonometric problems. It is all based upon the Mannheim-type rule.

Description of B. 91: This 81-page booklet is a mine of information concerning the types of graph papers and cloths now available. It is beautifully illustrated in color. All the usual types and many special ones are shown.

Description of B. 92: This 12-page booklet shows many interesting drafting aids which have some mathematical content.

Description of B. 93: This 254-page booklet is Part 1 of the 41st Edition of the K & E catalog. It lists and illustrates drawing and tracing papers and cloths, graph sheets, drawing instruments, drafting tools, printing and developing equipment, drawing boards and furniture, slide rules and other calculating instruments.

Appraisal of B. 90 through B. 93: In all of these booklets the clarity of the illustrations and the completeness of the coverage is outstanding. B. 90 should not be sent for unless a school has or intends to buy slide rules. In order to follow the instructions a slide rule should be in front of you. B. 91 is a real education in the ingenious and useful types of graph ma-

terial. B. 92 has some small amount of application, but may suggest some instruments which will improve the drawing of geometric plates. B. 93, the complete catalog, will deserve a great deal of study on the part of both teachers and pupils to see the equipment which is used to apply mathematics to the field of drafting and engineering.

CHARTS

C. 27—Road Maps of Industry

National Industrial Conference Board, 247 Park Avenue, New York 17, N. Y.

Graph charts; 8½″×11″; Free

Description of C. 27: These charts are really splendid examples of mathematical graphs on such subjects as U. S. Saving Bonds, Farm prices and parity, wholesale prices, military manpower, personal income, federal expenditures, world steel capacity, and gross market value of products of the United States. They are printed in clear and bright colors and are punched with three holes to put into notebooks. These charts are available free only to high school teachers, staff members of teachers colleges, and administrators at the above levels. They must be sent to school addresses. Sometimes additional copies of certain charts are available for pupil use.

Appraisal of C. 27: It is hard to overemphasize the attractiveness of these graphs; they are very suitable for mathematics classes since a variety of types are constantly used. The fact that they present the numerical facts of economics with up-to-date precision makes it mandatory that teachers use them to supplement the graphs which appear in textbooks. Even if they are merely displayed on bulletin boards they will improve the teaching of mathematics, but it would be a shame not to help the class to interpret them and find the many conclusions which can result.

EQUIPMENT

E. 46–E. 66—Grove's Moto-Math Set

BASIC

E. 46–(250-M)—Moto-Math Manual
E. 47–(250-1)—Blackboard Graph Chart
E. 48–(250-2)—Stainless Steel Split Pins
E. 49–(250-3)—Heavy Multi-colored Elastics
E. 50–(250-4)—Beam Blackboard Compass
E. 51–(250-5)—Two Semicircles—16 sectors
E. 52–(250-6)—Three Triangles
E. 53–(250-7)—Two Congruent Trapezoids
E. 54–(250-8)—Navigation Protractor
E. 55–(250-9)—Standard Protractor
E. 56–(250-10)—Combination Curve Drawing Instrument
E. 57–(250-11)—Straight Edge and Two Spring Clips With Cord
E. 58–(250-12)—Linkage Set
E. 59–(250-13)—Wooden Case for Accessories
E. 60–(250-14)—Miniature Fibre Airplanes

EXTRA

E. 61–(250-15X)—Hard Fibre Fractions Discs
E. 62–(250-16X)—30 Counting Discs
E. 63–(250-17X)—Abacus
E. 64–(250-18X)—Clock Face
E. 65–(250-19X)—Pantograph
E. 66–(250-20X)—Easel

Developed by Prof. J. Seth Grove, Head of Mathematics Department, State Teachers College, Shippensburg, Penna.
Purchased through Yoder Instruments, East Palestine, Ohio.

Description of E. 46—(250-M): This 30 page manual is designed to assist the teacher in the use of the Moto-Math Set. It presents many diagrams with descriptive instructions on how to use the material.

The manual lists demonstrations as follows: plane geometry—36, solid geometry—4, linkages—12, physics and mechanics—7, algebra—12, trigonometry—8, navigation—6, arithmetic—8, analytical geometry and calculus—18. Additional demonstrations can be devised by teachers and students. The price of the manual is 30¢ each—4 copies for $1.00.

Description of E. 47–(250-1): This blackboard graph chart is of 20 gauge sheet steel measuring 32″×34″ in a hardwood frame—ruled 1″ squares with holes punched at intersections of all lines and at other places for various conic section curves. There are 977 holes in all. This part of the equipment is the basic unit upon which the set is built. It is used in practically all demonstrations. The price of the graph chart is $35.00.

Description of E 48–(250-2): These 25 Split Pins are used in the holes on the Graph Chart to support various devices on the board and to serve as vertices of diagrams when elastic bands are used. The price for the set of 25 is $1.00.

Description of E. 49–(250-3): This set of 50 multi-colored elastics are of good rubber and serve the purpose of forming geometric figures on the graph board when stretched between the pins. The price for the 50 elastics is $.50.

Description of E. 50–(250-4): These compass arms are of flexible steel 12″ and 30″ long. An adjustable pin is provided, which makes it possible to mount the compasses on the blackboard graph. The price for the set is $3.00.

Description of E. 51–(250-5): Two semicircles, each of $\frac{1}{4}$″ composition material with glazed surface. Together they form a circle of 12″ diameter. Each semicircle is divided into 8 equal sectors, which are attached by leather at the circumference. These semicircles are used, when supported on the blackboard graph chart by pins and the ruler, to show area of a circle and its relation to the area of a parallelogram. The price for the set is $5.00.

Description of E. 52–(250-6): These triangles are constructed of $\frac{1}{4}$″ composition material, with white glazed surface, with inch rulings vertically and horizontally. The three triangles may be used separately and also can be assembled to form a rectangle or a parallelogram. They are 45-45-90, 30-60-90, and 75-60-45 triangles. They may be used with the blackboard graph chart as background when supported by the ruler. The price for the set is $4.00.

Description of E. 53–(250-7): These two congruent trapezoids are constructed of $\frac{1}{4}$″ composition material, with white glazed surface. They measure 8$\frac{1}{2}$″ on the base and 7″ high. They can be used separately or to form a parallelogram when put together to show area relationships and congruency. They may be used on the blackboard graph chart when supported by the ruler. The price for the set is $3.50.

Description E. 54–(250-8): This Navigation Protractor is of laminated paper, 7″ in diameter and has markings around its circumference in units of 5°. It may be mounted on the blackboard graph chart to illustrate angle size especially in solving navigation problems. No. 250-14, the miniature airplanes, may be used with this protractor to make the diagrams more realistic. The price for the protractor is $.75.

Description of E. 55–(259-9): This is a standard circular protractor on a masonite base with a 7″ diameter. It has markings in units of one degree from 0 through 360 each way. It can be mounted on the blackboard graph chart to illustrate angle size. It can be used to draw cycloids and trochoids on the blackboard graph chart by using the proper holes for chalk and by revolving the protractor on the ruler. The price of this protractor is $2.50.

Description of E. 56–(250-10): This set consists of pins, cord of different lengths, a chalk holder, and a special wooden ruler in the form of a T square to be mounted on the ruler on the blackboard graph chart. The set is used for drawing the parabola, ellipse and hyperbola. The price for the set is $3.50.

Description of E. 57–(250-11): This is the straight edge ruled in inches by units of $\frac{1}{8}$ on one side and metric units on the other. It is approximately 30 inches long and is used for measuring, for mounting on the blackboard graph chart by means of 2 spring clips which come with it, or for drawing lines. When mounted on the

blackboard graph chart, it is useful for supporting many of the objects used in a variety of demonstrations. The price for the set is $1.75.

Description of E. 58–(250-12): This linkage set consists of 12 strips of hard white fibre ⅜″ wide and in varying lengths, 1 piece 14½ inches, 4 pieces 12 inches, 2 pieces 8 inches and 2 pieces 6 inches. It is accompanied by 6 split pins, 6 brass nails, and 6 rubber spacers and a chalk holder. The strips may be used to demonstrate linkages on the blackboard graph chart in problems such as: If the base of a triangle is 12″ and the altitude is 8″, what are the limits in the length of the other two sides? Many locus principles can also be demonstrated. The price for the set is $4.50.

Description of E. 59–(250–13): This is the wooden case measuring 9″×33″×2½″. It is divided into 3 compartments and is used to store and transport the many items that make up the total equipment for the set. The price is $2.50.

Description of E. 60–(250-14): There are three white miniature airplanes measuring 4″×4″ made of hard fibre with holes for mounting on the blackboard graph chart. They are used to make problems in Navigation more realistic. The price for the set is $1.50.

Note on prices: If the individual prices for items 250-M through 250-14 are added, the total price would be $70.00. However, the complete set may be purchased for $60.00.

Extra Equipment—Sold Separately

Description of E. 61–(250-15X): These hard-fibre fraction discs consist of two circles, a red one and a white one, mounted together so that they may be rotated with respect to each other. When mounted at the center of the board all useful fractions may be quickly illustrated by rotation. The price is $2.50.

Description of E. 62–(250-16X): This set of counting discs consists of 30 pieces of white hard fibre 1½″ in diameter with a hole punched in the center for mounting purposes. Three spring clips are provided on which to assemble the discs. They may then be mounted on the chart to serve as objective material for teaching the meaning of number and the fundamental facts and processes of arithmetic. For example, showing 3 groups of 2 in the number 6. The price is $2.00.

Description of E. 63–(250-17X): This Abacus consists of three stainless steel strands each with 10 colored wooden beads. They may be mounted on the blackboard graph chart to teach the meaning and use of number. The beads can be slid up and down the wires. Each bead is constructed so as to remain in any position on the wires even when the wires are in a vertical position as illustrated. Each individual string of ten beads is easily removable from the abacus so that it can be used for more elementary work with numbers before the pupil is ready for the use of the abacus as such. The price is $4.00.

Description of E. 64–(250-18X): This clock face is plastic laminated cardboard 12″ in diameter with two steel hands. It may be mounted on the Graph Board and has many uses in teaching number and time. The price is $2.00.

Description of E. 65–(250-19X): The pantograph consists of four wooden (maple) bars (2–18″ and 2–20″) fitted to mount on the Graph Board. It is a mechanical instrument used to enlarge or reduce drawings, pictures and diagrams. With it comes pins, eyelets and a metal base for mounting purposes. The price is $3.00.

Description of E. 66–(250-20X): This is the hardwood easel which has three legs and a 34″ chalk tray. It is used to support the Graph Board. The price is $15.00.

Appraisal of E. 46–(250-M): The manual contains 30 pages and 112 key problems with illustrative and explanatory material on the use of the Moto-Math Set. By following its directions and observing the diagrams, the equipment can be effectively used to carry out the many demon-

strations listed in plane geometry, solid geometry, linkages, physics and mechanics, algebra and graphs, trigonometry, navigation, analytic geometry, calculus, and arithmetic. The diagrams, showing the use of the equipment on a graph background, are placed beside the explanatory material for convenience. Teachers and students will find the manual very helpful in carrying out the demonstrations. Additional problems with illustrations on the subjects listed and other subjects will be issued from time to time.

Appraisal of E. 47–(250-1): The blackboard graph chart is sturdily built, attractive, and will stand plenty of use. It may be hung by the two strong hooks provided or supported by the easel. It is a convenient and easy-to-use piece of equipment and can add much when used for class demonstrations or by individual pupils. It is a good blackboard graph that can be used for many mathematical purposes.

Appraisal of E. 48–(250-2) and E. 49–(250-3): The split pins and elastics are necessary pieces of equipment to carry out many of the demonstrations. The pins are well made of stainless steel and will stand up well. The elastics are of good rubber and add color to the demonstrations. It might be better to have elastics of two lengths—some being longer than those now provided.

Appraisal of E. 50–(250-4): These special compasses are invaluable in drawing circles on the Graph Board. A circle may be drawn at any point by inserting the split pin of the "center post" in any desired hole. A slight pressure on the cap of the "center post" releases the "compass arm" which may then be adjusted to any desired length, 1″ to 12″ with the short arm and 1″ to 30″ with the long arm.

Appraisal of E. 51–(250-5), E. 52–(250-6), E. 53–(250-7): The semicircles, triangles and congruent trapezoids are well made of composition material and are valuable aids when used either with or without the Graph Board for many demonstrations of area and linear relationships.

Appraisal of E. 54–(250-8) and E. 55–(250-9): The navigation and standard circular protractors are useful and valuable aids when used with the Graph Board for demonstrations involving angle size. When used with the colored elastics and the pins the protractors mounted on the graph board make an attractive and functional device. The protractors would probably be even more effective if they were slightly larger so that they could be easily seen from the back of a classroom.

Appraisal of E. 56–(250-10) and E. 57–(250-11): The set of pins, cord and chalk holder and the straight edge are indispensable pieces of equipment for drawing lines and measuring. They are well made and should last a long time. The straight edge has the additional value of its use for supporting other objects when attached to the graph board.

Appraisal of E. 58–(250-12): The linkage set is a valuable addition to the equipment since many area and linear relationships can be clearly demonstrated when the strips are used on the graph board. Many loci using linkage strips can be quickly and clearly demonstrated. The importance of this subject of linkages is often overlooked in the teaching of mathematics. This method of demonstrating them makes it a pleasant and worthwhile project. With the equipment provided, the strips are easily attached to the graph board.

Appraisal of E. 59–(250-13): The wooden case is a must since the many pieces of equipment should be carefully stored when not in use. The case is well built and worth the money.

Appraisal of E. 60–(250-14): These miniature planes are not indispensable, but do add a touch of realism when used on the graph board in navigation problems involving drift angle, heading and course. They are well-made of hard fibre and should last a long time.

Appraisal of Extra Equipment—E. 61 through E. 66–(250-15X through 250-20X): This equipment has many uses in demonstrations of various kinds. The fraction

discs, counting discs, abacus and clock face are valuable aids, especially at the elementary school level, for objective material in the teaching of number, the meaning of the fundamental facts, and the meaning of the fundamental processes of arithmetic. They can be used separately, but are more effective when used with the graph board as background. The pantograph is a helpful piece of equipment that is well-made and comes complete with the necessary parts. It is useful for enlarging or reducing drawings, pictures and diagrams.

There are many other uses for the set not listed here in a variety of subjects. It would, for example, be helpful to the arithmetic teacher in drawing different kinds of graphs, showing perimeters and areas, illustrating the meaning of per cent, decimals and fractions, as well as in making diagrams in teaching understanding in the solution of problems. (Reviewed by John K. Moulton and Ralph F. Ward of Brookline High School, Brookline, Mass.)

FILMS

F. 21A—What Is Money?*

Producer: Coronet Films, Coronet Building, Chicago 1, Ill.

Educational Collaborator: Paul L. Salsgiver

16 mm. film; Black and white; Sound; one reel; nine minutes; $45.00; colored, $90.00.

Description: Recommended grades from producer: Intermediate, Junior High, Senior High, and Adult.

In this film we follow the journey of a brand new five dollar bill through its many transactions. The bill starts in circulation with Tom receiving the five dollar bill from his father. He is to buy a gallon of paint with it at the hardware store. From the hardware store the five dollar bill goes to an electrician in payment for special services he has rendered. On the way home the electrician stops in at a gasoline station to get some gas and pays for it with the five dollar bill. Mrs. Moore comes in the gasoline station to cash a check and receives the bill as part of the transaction. After cashing her check, Mrs. Moore goes downtown to buy a pair of bookends which cost five dollars. The store is then shown paying off its employees and the five dollar bill goes to one of the employees who in turn takes it to a radio store. This employee had bought a radio on the installment plan and wants to make her weekly payment. The radio man upon receiving this bill gives it to a boy who has been working for him. We see that this boy is Tom, the same boy who started the five dollar bill on its journey. Tom deposits the money in the bank.

Interwoven among the journeys of the five dollar bill are pictures of how the barter system was used in ancient times and still is today. It shows two boys, one is trading a bicycle light for a catcher's glove. We also see how different articles have been used as money such as: spears, shells, and wampum. We are shown how people used to make their own money and how the government now makes it.

The film closes with a summary of the uses of the five dollar bill. First, it was a medium of exchange for goods; second, it was a medium of exchange for services; third, it was a standard of value; fourth, it was a standard of future payments; and fifth, it was a storehouse. It is shown that the bill could conceivably have gone to a world traveler and traveled all over the world.

Appraisal: Seven teachers evaluated this film and five of the teachers in considering the entire film said it was good for introducing new material, providing a common experience, and for motivation. All of the teachers thought the film was good for review. The grade level was placed between the Elementary and College level with the 7th and 8th grade being checked by 53%. One half of the evaluators favored arithmetic as the course best suited while the other half chose consumer mathematics. One teacher thought the

(*Continued on page 584*)

* This is the first aid to teaching which we have reviewed for the second time. Letters will follow the numbers to indicate repeat reviews.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* JOHN A. BROWN
The University of Wisconsin, Madison, Wisconsin

IN ADDITION to statistical information on school offerings, in this department during this year, there will be published accounts of interesting and unusual school activities which have been contributed directly to *The Mathematics Teacher*, or which have been brought to the attention of the editors in other ways.

FOURTH TERM ALGEBRA

The topic at one of the discussion groups of the summer session of the National Council of Teachers of Mathematics at Northfield, Minnesota, was "What Belongs in Fourth Term Algebra?" The variety of opinions and practices prompted several of the group to suggest that a report in *The Mathematics Teacher* might be a step in determining whether other teachers would be interested in further discussion at a subsequent meeting.

At least two factors are responsible for the wide variety of topics taught in fourth term high school algebra. One is the time the fourth term is taught. Some schools teach algebra the ninth and tenth years; others the ninth and eleventh years; still others the ninth, one semester of the eleventh, and one semester of the twelfth year. Another factor is the purpose of the course: a polishing up of previous work in algebra, a preparation for algebra in college, an introduction to the topics taught in algebra in college, or some combination of these purposes.

In our discussion we began with the list of topics offered in a widely used textbook, attempted to determine their value to a high school student, and tried to decide whether they belonged in fourth term algebra. By a show of hands we made a record of the practices in the schools represented. It must be remembered that the time of giving the course is often a deciding factor in choosing topics. For instance, if fourth term algebra follows trigonometry, obviously logarithms must be taught at another time.

The following topics, some of them continuations of third term work, were the only ones on which there was majority agreement: factoring, fractions, exponents, radicals, imaginaries, equations linear and quadratic in one and two variables, equations involving radicals, equations of degree higher than the second that can be solved with the factor theorem.

The topics taught by some of the teachers in the group, but not by a majority of them were: functions and their graphs, determinants in two and three variables, ratio, proportion, variation, arithmetic and geometric progressions, logarithms, permutations, combinations, and probability.

No one in the group did anything with analytical geometry or calculus and the college teachers present agreed that these topics were best left for college students.

Do you think that further investigation of the subject would be profitable?

ONA KRAFT
Collinwood High School
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NEWARK SECONDARY SCHOOLS SPONSOR MATHEMATICS FAIR

Come to the Fair! Come one! Come all! On April 25, 1951 Arts High School in Newark, New Jersey, was the scene of this unusual hubbub sponsored by the Newark Council of Teachers of Mathematics. Had you entered the school on that afternoon large posters would have invited you to

the "fair grounds". More pointedly, the decimals of π strung high on a wire would have directed you to the gym.

However you wouldn't have recognized the gym. The centerpiece, a huge solid of many faces, used a few of them to observe you as you pinned on the name tags handed to you by the "barkers". Posters of geometric designs in thread and wool tacked on large screens, and mathematical newspaper clippings adorning the walls, served as open invitations to the sidelines and byways of mathematics. These led next to the large tables displaying geometric models and applications and attended by students of Newark High Schools; students of Montclair State Teachers College; Mr. George Kays, an instructor at the College; Mr. Charles Goeller, an instructor at the Newark School of Fine and Industrial Arts; and various other teachers of Newark Secondary Schools.

At many of the tables you would have been forced to edge your way closer in order to get a good look at the soap film experiments, the statistical array of shot in columns, the surveying instruments, the various models of solids—some in paper form, others of ivory texture. Once aroused, your curiosity would not have been satisfied until you circled the fair grounds again and again, brushing shoulders here with young mathematics students, there with teachers, and perhaps even with a celebrity or two, like Mr. Michael McGreal, Assistant Superintendent of Schools of Newark, or Dr. Virgil S. Mallory, Head of Mathematics Department at Montclair State Teachers College, who found time in their busy schedules to meet us at the Fair. But once again the "barkers" were at work urging everyone on to the "big tent". On the way you must stop for your free refreshments, in the form of lollipops, and a "program", in the form of a mimeographed numbered pamphlet containing mathematics puzzles, problems, and diagrams.

In the auditorium Harold A. Gouss of Newark Central High gave a short welcoming address in his official capacity as president of the sponsoring organization. Next, the Ringmaster, Mr. Ernest R. Ranucci, Chairman of the Mathematics Department of Weequahic High and of the program committee for the Fair, introduced the main "acts".

First, Sanford Kahn, student of Weequahic High gave his magic demonstration of soap film experiments. Next Irving Eng, student of Arts High showed the use of the abacus in computation. Mr. Goeller then spoke on his process for constructing geometric models with artistic overtones. The final act guided us on a historical journey through the development of units of measurement and was led by a student of Montclair State Teachers College.

Mr. Max Sobel, teacher at Robert Treat Junior High, took over the main circle to introduce the final event of the Fair, a contest of mathematical skill. He determined five contestants by a drawing of numbers based on those of the program pamphlets. Peter Tango of Cleveland Junior High was adjudged the winner and awarded the grand prize, a 10″ slide rule accompanied by 10 humorous rules for happiness via its use.

In the Executive Council of the Newark Council of Teachers of Mathematics the usual post-mortems helped adjust the financial balance of this event from its physical debit side to its intangible benefits to students and teachers on the credit side. We teachers of mathematics in Newark had, in the past year, seen fit to band together to discuss our problems and attempt concerted efforts at their solution; to stimulate interest in mathematical education on all levels, administrative, teacher, student, and parent; to do so vividly and dramatically, not disdaining the well-proven methods of the salesman, the "barker", to achieve these worthwhile ends. Therefore we felt justified in our estimate of the credits accruing to the organization from this first public project. We were not unmindful of the wonderful

cooperation received from the administration at Arts High, from the Mathematics Department of Montclair State Teachers College and from all of the teachers and students who lent a hand in this achievement. In fact it was this very cooperation which has greatly encouraged us in contemplating plans for future activities based on the main objectives previously noted.

Perhaps others in your organization who feel a need for cooperative action on a local level will also be encouraged to join us in achieving them on a wider scale. Surely this will make for a more vital mathematical program by teachers in times when everything is in great need of vitality and nothing more so than the field of mathematical education in the secondary schools.

HAROLD A. GOUSS, President
The Newark Council of Teachers of Mathematics
Central High School
Newark, New Jersey

Aids to Teaching

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speed of development of ideas and duration of scenes was too fast. No inaccuracies were found in the mathematical content. 57% said the film would hold the interest of the student and that the teaching methods in the film were conducive to learning. 86% thought that the content could not be just as effectively and efficiently presented in some other way. Six of the seven teachers said they would use the film in their classroom. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway)

F. 65—Angles

Knowledge Builders, 625 Madison Avenue, New York 22, N. Y.

16 mm. film; Black and white; Sound; one reel; eleven minutes; $40.00.

Description: This depicts various types of angles and their relationship to each other.

It is shown that two rays intersecting form four angles. Complementary and supplementary angles are defined and illustrated by animation. The bisector of an angle is defined and a circle with an angle of sixty degrees marked off. This is divided into two thirty degree angles to illustrate the bisecting of the angle. A drawing is made of a transversal through two parallel lines, and it is then pointed out that there are four acute and four obtuse angles as a result of this. Interior and exterior angles are defined and pointed out.

Various scenes such as buildings, bridges, etc., are shown throughout the film to illustrate the practical use of angles.

Appraisal: For the evaluation of this film there were thirty-seven teachers of mathematics present. Taking the entire picture into consideration 83% thought it was fair or poor for introducing new material, 83% said it was good or fair to augment explanations, 44% checked poor for the development of skills, and 35% said it was excellent for review. 57% favored grade 10 for this film while the other evaluators checked grades 7 through the college level. 75% checked Geometry as the course best suited for this film while 15% checked Shop Mathematics. Nine teachers thought that the speed of development of ideas and duration of scenes was too fast. 38% found inaccuracies in the film. Two-thirds of the evaluators thought the film would partially hold the interest of the students, but 80% found the teaching methods conducive to learning. 50% found that the photography and sound was good. One-half of the teachers thought that the amount of material was not reasonable for comprehension in one

(*Continued on page 588*)

REFERENCES FOR TEACHERS

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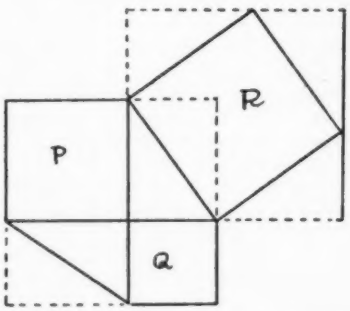
The Theorem of Pythagoras

JUSTLY called the most celebrated theorem of geometry, the Pythagorean proposition is probably the one bit of mathematics which millions of laymen will remember long after they have forgotten whatever other mathematics they may once have known. The theorem is notable first because of the rich historical associations with which it is attended; secondly, because of the amazing variety of proofs which have been given; and thirdly, because further exploration quickly leads to interesting and unexpected byways, such as the Golden Section, dynamic symmetry, logarithmic spirals, angle trisection, duplication of the cube, squaring the circle, determining the value of π, the concept of irrational number, regular and star polygons and polyhedra, theory of numbers, constructibility of angles and polygons, continued fractions, phyllotaxy, musical scales, Diophantine equations, Heronian triangles, and Pythagorean number lore.

Thus the theorem of Pythagoras lends itself admirably as a topic for discussion in mathematics clubs or for further study by mathematically minded students. Many proofs are readily accessible, particularly in the well known monograph by Loomis and in recent volumes of *Scripta Mathematica*. Some of these analyses are challenging to able minds, as for example, the alleged proof of *Multatuli* (1867), than which, the author asserted, no other proof could possibly be simpler. In view of these observations, teachers may find the following references of more than passing interest.



MULTATULI'S "PROOF": $P+Q=R$

1. PROOFS AND DISCUSSIONS

Baravalle, Hermann. "A Dynamic Proof in a Succession of Five Steps." *Scripta Mathematica*, 1947, 13: 186; also, *National Council of Teachers of Mathematics*, 18th Yearbook, 1945, pp. 80–81.

Baravalle, Hermann. "A Model for Demonstrating the Pythagorean Theorem." *Scripta Mathematica*, 1950, 16: 203–207.

Blakslee, T. "Ptolemaic and Pythagorean Theorems, from an Identity." *School Science and Mathematics*, 1914, 14: 748.

Colburn, A. "Pons Asinorum; New Solutions of the Pythagorean Theorem." *Scientific American Supplement*, December 1910, 70: 359; 382–383.

Colburn, A. "Study of the Pythagorean Theorem and Its Proofs." MATHEMATICS TEACHER, 1911, 4: 45–47.

Condit, A. "New Proof of the Pythagorean Theorem." *School Science and Mathematics*, 1940, 40: 379–380.

Davidson, E. "High School Boy's Proof of the Pythagorean Theorem." *School Science and Mathematics*, 1907, 7: 777–778.

Dix, R. H. "Pythagoras' Theorem." *Mathematical Gazette*, 1945, 29: 70.

Diemel, R. F. "Pythagorean Theorem." *Science*, 1911, n.s. 33: 457.

Eagle, Edwin. "Pythagoras and Ptolemy Must Have Looked at the Conclusion." *The Pentagon*, 1951, 10: 79–83.

Evans, G. "Proof of the Pythagorean Theo-

rem." MATHEMATICS TEACHER, 1923, 16: 440.

Georges, J. "Pythagorean Theorem." *School Science and Mathematics*, 1927, 27: 367–378.

Goldman, Bernard. "A Proof of the Theorem of Pythagoras." *School Science and Mathematics*, 1943, 43: 781–782.

Hartig, H. "Proof of the Pythagorean Theorem." *School Science and Mathematics*, 1913, 13: 819.

Joffe, J. A. "Old and New Proofs of the Pythagorean Theorem." *Scripta Mathematica*, 1948, 14: 127.

Jones, Phillip. "The Pythagorean Theorem." MATHEMATICS TEACHER, 1950, 43: 162–163.

Kailasamaiyer, N. "Proof of Pythagoras' Theorem." *Mathematical Gazette*, December 1944, v. 28, Mathematical Note #1746.

Katanik, H. "New Proof of the Pythagorean Theorem." *School Science and Mathematics*, 1915, 15: 669.

Kinney, J. M. "New Proofs of the Theorem of Pythagoras." *School Science and Mathematics*, 1941, 41: 249–254.

Knoer, A. "Proof of the Theorem of Pythagoras." MATHEMATICS TEACHER, 1925, 18: 496–497.

Langman, Harry. "A Proof Resulting from the Solution of a Simple Dissection Problem." *Scripta Mathematica*, 1948, 14: 16.

Lawrence, B. E. "Pythagoras and an Extension." *Mathematical Gazette*, December 1944, v. 28, Mathematical Note #1751.

McCarthy, J. P. "Huygens' Proof of the Theorem of Pythagoras." *Mathematical Gazette*, 1936, 20: 280–281.

McFarlane, A. "Pythagorean Theorem." *Science*, 1911, n.s. 34: 181–182.

Northrup, E. "Is This a Dynamical Proof of the Pythagorean Theorem?" *Science*, 1910, n.s. 32: 863–864.

"Old and New Proofs of the Pythagorean Theorem." *Scripta Mathematica*, 1946, 12: 266; 1947, 13: 116.

"Pythagorean Theorem." *Science*, 1911, n.s. 33: 457.

Raghava Rao, K. V. "Proof of the Pythagorean Theorem." *Scripta Mathematica*, 1950, 16: 168.

Scott, W. "Pythagorean Theorem." *School Science and Mathematics*, 1910, 10: 550.

"Solving the Theorem of Pythagoras." *Scientific American Supplement*, 1917, 84: 362.

2. PYTHAGOREAN NUMBERS: INTEGRAL AND RATIONAL RIGHT TRIANGLES

Aude, H. T. R. "Rational Right Triangles." *American Mathematical Monthly*, 1932, 39: 353–354.

Ball, W. W. R. and Coxeter, H. S. M. *Mathematical Recreations and Essays* (11th Edit.) New York, Macmillan, 1942; Rational Right-Angled Triangles, pp. 57–59.

Ballentine, J. P. and Brown, O. E. "Pythagorean Sets of Numbers." *American Mathematical Monthly*, 1938, 45: 298–301.

Block, D. and Umansky, H. L. "Pythagorean Variations." *Scripta Mathematica*, 1949, 15: 243–244.

Brown, E. N. "Integral Right Triangles." *School Science and Mathematics*, 1941, 41: 799–800.

Brown, F. "Formulae for Integral Sided Right Triangles." *School Science and Mathematics*, 1934, 34: 21–25.

Cameron, W. T. "Pythagorean Integers." *Australian Mathematics Teacher*, Nov. 1946, v. 2, Mathematical Notes, #45.

Carosh, M. "On the Equation $x^2+y^2=z^2$." *American Mathematical Monthly*, 1939, 46: 228+.

Clarke, J. H. C. "Pythagorean Integers." *Australian Mathematics Teacher*, Nov. 1946, v. 2, Mathematical Notes, #44.

Cohen, Israel and Jones, Phillip S. "Rational Sets of Pythagorean Numbers." MATHEMATICS TEACHER, 1950, 43: 352.

Colwell, L. "Exploring the Field of Pythagorean Number." *School Science and Mathematics*, 1940, 40: 619–627.

Dickson, L. E. "Rational-Sided Triangles." *American Mathematical Monthly*, 1894, 1: 6–11.

Dickson, L. E. "Rational Triangles and Quadrilaterals." *American Mathematical Monthly*, 1921, 28: 244–250.

Dresden, A. *An Invitation to Mathematics*. New York, Henry Holt and Co., 1936; Pythagorean Numbers, pp. 1–11.

Duncan, D. C. "Generalized Pythagorean Relationships." *National Mathematics Magazine*, 1936, 10: 209–211.

"Formulae for Rational Right Triangles." *School Science and Mathematics*, 1910, 10: 683; 1911, 11: 293.

Ginsburg, Jekuthiel. "Complex Numbers as Generators of Pythagorean Triangles." *Scripta Mathematica*, 1947, 13: 105.

Ginsburg, Jekuthiel. "The Generators of a Pythagorean Triangle." *Scripta Mathematica*, 1945, 11: 188–189.

Ginsburg, Jekuthiel, "Pythagorean Pleasantries." *Scripta Mathematica*, 1945, 11: 191.

Goodrich, M. T. "A Systematic Method of Finding Pythagorean Numbers." *National Mathematics Magazine*' 1945, 19: 395–397.

Goodstein, R. "Rational Triangles." *Mathematical Gazette*, 1939, 23: 264–267.

Heath, T. *A History of Greek Mathematics*. Vol. 1. London, Oxford-Clarendon Press, 1921; Pythagorean Numbers, pp. 79–82.

Kraitchik, M. *Mathematical Recreations*. New York, W. W. Norton and Co., 1942; Pythagorean and Heronian Triangles, pp. 95–108.

Lehmer, D. N. "Rational Triangles." *Annals of Mathematics*, 1899–1900, Ser. 2, v. 1, pp. 97–102; also, *American Journal of Mathematics*, 1900, 22: 38.

Martin, Artemas. "Formulae for Rational Right Triangles." *School Science and Mathematics*, 1911, 11: 293–294.

Martin, Artemas, "Groups of Rational Right-

Angled Triangles." *Scripta Mathematica*, 1948, 14: 33–34.

Martin. Artemas. "On Rational Right Triangles." *Proceedings of Fifth International Congress of Mathematicians*, v. 2, Cambridge; 1913.

McCarthy, J. P. "Rational Sided Triangles." *Mathematical Gazette*, 1936, 20: 152.

McLean, E. "Pythagorean Numbers." *Mathematical Gazette*, 1940, 24: 59; 125.

Merriman G. M. *To Discover Mathematics*. New York, John Wiley and Sons, 1942; Pythagorean Numbers, pp. 42–48.

Ore, O. *Number Theory and Its History*. New York, McGraw-Hill Book Co., 1948; Pythagorean Numbers, pp. 165–179.

Osborne, G. A. "A Problem in Number Theory." *American Mathematical Monthly*, 1914, 21: 148–150.

Petrie, F. "Integral Right-angled Triangles." *Nature*, Sept. 1933, 132: 411, 597; 133: 106.

Piza, Pedro. "Pythagorean Triangles by Sums of Powers." *Scripta Mathematica*, 1949, 15: 90.

Raine, C. W. "Pythagorean Triangles from the Fibonacci Series 1, 1, 2, 3, 5, 8, 13, 21, etc." *Scripta Mathematica*, 1948, 14: 164–165.

"Rational Right Triangles." *American Mathematical Monthly*, 1900, 7: 232–233; 271.

Robinson, L. V. "Building Triangles with Integers." *National Mathematics Magazine*, 1943, 17: 239–244.

Ross, W. B. "A Chart of Integral Right Triangles." *Mathematics Magazine*, 1949, 23: 110–114.

Shedd, C. L. "A Formula for Primitive Pythagorean Triangles." *Scripta Mathematica*, 1943, 9: 268.

Talbot, W. R. "Pythagorean Triples." *American Mathematical Monthly*, 1949, 56: 402.

Whitlock, W. P. "A Family of Giant Pythagorean Triangles." *Scripta Mathematica*, 1939, 6: 246.

Whitlock, W. P. "An Interesting Classification of Pythagorean Triangles." *Scripta Mathematica*, 1943, 9: 268.

Whitlock, W. P. "Pythagorean Variations." *Scripta Mathematica*, 1946, 12: 259–265.

Wright, H. N. *First Course in the Theory of Numbers*. New York, John Wiley and Sons, 1939; pp. 92–96.

Young, J. W. A. *Monographs on Topics of Modern Mathematics*. London, Longmans, Green and Co., 1911; pp. 316–319.

3. Special Cases of Rational Right Triangles; Heronian Triangles

Allison, N. *Mathematical Snack Bar*. New York, Chemical Publishing Co., 1936; Heronian Triangles, pp. 116–125.

Anema, A. S. "Pythagorean Triangles with Equal Perimeters." *Scripta Mathematica*, 1949, 15: 89.

Bradley, H. C. "Rational Oblique Triangles." *American Mathematical Monthly*, 1923, 30: 70.

Brown, D. M. "Numerical Double-Angle Triangles." *The Pentagon*, 1948, 7: 74–80.

Cheney, W. F. "Heronian Triangles." *American Mathematical Monthly*, 1929, 36: 22–28.

Dickson, L. E. "Rational Triangles and Quadrilaterals." *American Mathematical Monthly*, 1921, 28: 244."

Herzog, F. "Pythagorean Triangles with Equal Perimeters." *American Mathematical Monthly*, 1949, 56: 32+.

Kasner, Edward. "Neo-Pythagorean Triangles." *Scripta Mathematica*, 1947, 13: 43–47.

Landes, Leo. "On Equiareal Pythagorean Triangles." *Scripta Mathematica*, 1945, 11: 97–99.

Martin, Artemas. "Groups of Rational Right-Angles Triangles Whose Hypotenuses are Consecutive Numbers Less than 1000." *Scripta Mathematica*, 1948, 14: 33–34.

Martin, A. "Properties of Rational Plane Triangles Whose Sides Are Whole Numbers." *School Science and Mathematics*, 1913, 13: 32–36.

Miksa, F. L. "Pythagorean Triangles with Equal Perimeters." *Mathematics Magazine*, 1950, 24: 52–53.

Roberts, H. C. "Pythagorean Triangles and the Tarry-Escott Problem." *Scripta Mathematica*, 1950, 16: 132–133.

Shedd, C. L. "A Hypotenuse Common to 64 Primitive Right Triangles." *Scripta Mathematica*, 1949, 15: 132.

Struyk, Adrian. "The Generation and Use of Heronian Triangles." Mathematics Teacher, 1951, 44: 264–265.

Umansky, H. L. "A Triangle of Pythagorean Hypotenuses." *Scripta Mathematica*, 1950, 16: 128.

Whitlock, W. P. "Pythagorean Triangles in Arithmetical Progression." *Scripta Mathematica*, 1946, 12: 293.

Whitlock, W. P. "Pythagorean Triangles with a Given Difference or Sum of Sides." *Scripta Mathematica*, 1945, 11: 75–81.

Whitlock, W. P. "Pythagorean Triangles with Square Perimeters." *Scripta Mathematica*, 1948, 14: 60–61.

Whitlock, W. P. "Rational Right Triangles with Equal Areas." *Scripta Mathematica*, 1943, 9: 155–161; 265–267.

4. Pythagorean Theorem in Design

MacMahon, P. A. "Pythagoras's Theorem as a Repeating Pattern." *Nature*, 1922, 109: 479–579.

Schorer. "Tapetenmuster und der Satz des Pythagoras." *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht*, 1929, 60: 434–439.

5. Books and Monographs

Arnoux, G. *Le cas général du carré de l'hypothenuse*. Digne, 1889.

Hoffmann, J. J. I. *Der pythagoräische Lehrsatz mit 32 Beweisen*. Mainz, 1819, 1821.

Lietzmann, Walther. *Der pythagoreische Lehrsatz, mit einem Ausblick auf das fermatsche Problem*. Leipzig, Teubner, 1912. 72 p.

Lietzmann, Walther. *Von der pythagoreischen*

Gleichung zum fermatschen Problem. Leipzig and Berlin, Teubner, 1937. 48 p.

Loomis, Elisha. *The Pythagorean Proposition.* Berea, Ohio, Baldwin-Wallace College, 1941. 214 p. Copies may be obtained from Elatus G. Loomis, 1319 West 111 Street, Cleveland 2, Ohio.

Müller, J. W. *Systematische Zusammenstellung der wichtigsten bisher behandelten Beweise des pythagoräischen Lehrsatzes.* Nürnberg, 1819.

Naber, H. A. *Das Theorem des Pythagoras; wiederhergestellt in seiner ursprünglichen Form und betrachtet als Grundlage der ganzen Pythagoreischen Philosophie* Haarlem, Verlag von P. Visser Azn., 1908. 239 p

Ruper, W. W. *Famous Geometrical Theorems and Problems with Their History* Boston, D. C. Heath & Co., 1900

Thomas, K. *Das pythagoräische Dreieck und die ungerade Zahl.* Berlin, Herbig, 1859.

Wipper, G. *Sechsundvierzig Beweise des pythagoreischen Lehrsatzes, nebst kurzen biographischen Mitteilungen über Pythagores.* (Trans. from the Russian of F. Graup). Leipzig, H. Barsdorf, 1880.

Aids to Teaching

(*Continued from page 584*)

showing, and 97% said that the content could be just as effectively and efficiently presented in some other way. No distinction was made between "line" and "line-segment" and the definition of "adjacent angle" was not clear. The general comment seemed to be that the film definitely had many weak points. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway.)

F. 66—Polygons

Knowledge Builders, 625 Madison Avenue, New York 22, N. Y.

16 mm. film; Black and white; Sound; one reel; twelve minutes; $40.00.

Description: The film opens by showing real life polygons such as an air view; cut stones; honeycombs, which have the hexagon as the favorite design; and snowflakes, which are generally six-sided polygons.

The definition of a polygon is given and the ideas of convex and concave are clarified by animation. Pictures of irregular quadrilaterals, isosceles trapezoids, the rectangle and parallelogram are shown. A figure is then depicted with three sides, it is then changed to four sides, five sides, six sides and on up to twelve sides. By animation it is shown that as the sides are increased the polygon becomes more and more like a circle and that an infinite number of sides forms a circle. The terms "equiangular," "equilateral," and "regular" polygons are defined.

It is shown that in a regular polygon (a six-sided figure is used) the perpendicular bisectors of two sides meet in a point which is the center. The radius of a circumscribed and inscribed circle is explained. A description of the inscribed circle and the circumscribed circle of a regular polygon is given.

The sum of the interior angles theorem and the sum of exterior angles theorem are covered in detail.

Appraisal: There were twenty-five teachers present at the evaluation. Taking the entire film into consideration 90% thought the film was good for providing a common experience, 72% checked it was good to augment explanations, and 60% said it was good to introduce new material, to develop skills, to motivate and to review. 88% of the evaluators checked grades nine through eleven as best suited for the film, while 54% favored grade ten. The film is especially suited to Geometry with four teachers checking Shop Mathematics. Four teachers thought the speed was too fast for the development of ideas and duration of scenes, while one thought the speed was too slow. Four evaluators found inaccuracies in the film. 91% of the teachers said they would use this film in their classroom. The general comment was that the film failed to distinguish between essence and consequence in places. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway)

Correction: I.34—*Slide Rule and Mathematical Tables* should not be ordered from Standard Transformer Corp., Chicago. It may be secured from distributors of electronic parts such as Allied Radio Corp., 833 W. Jackson Blvd., Chicago 7, Illinois for 98 cents.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
135-12 77*th Avenue, Flushing* 67, *N. Y.*

MANY teachers of mathematics believe that a mathematical recreation is a species of its own. In other words, not all mathematical situations may be turned into mathematical recreations. This department proposes to explode this misconception once and for all. Any mathematical situation, if appropriately treated, may become a mathematical recreation. While "exploding" the above misconception, this department will reveal some of the "secrets of the trade" so that any mathematics teacher may become a creator of mathematical recreations.

The program for the creation of mathematical recreations is comparatively simple. . . . Its simplicity is predicated on two relatively important factors. One, a sense for the incongruous. The second, a slight affliction with imaginativeness. These two, if practiced properly, and, if employed timely, should enable any teacher to turn a mathematical situation into an interesting pastime. We shall illustrate the above "propositions" with a few examples.

Consider the following geometric original. *Every vertex of a parallelogram is joined by straight lines with the midpoints of the opposite sides. What portion of the area of the parallelogram is bounded by these straight lines?*

We shall defer the proof of this original for the time being. Suffice it to say that the bounded area will be equal to one-sixth of the area of the parallelogram.

Now, let us consider what facts are here before us. These are tabulated below.

(*a*) A parallelogram and its area.
(*b*) The midpoints of the sides of the parallelogram.
(*c*) Lines joining the vertices of the parallelogram with the midpoints of the opposite sides.

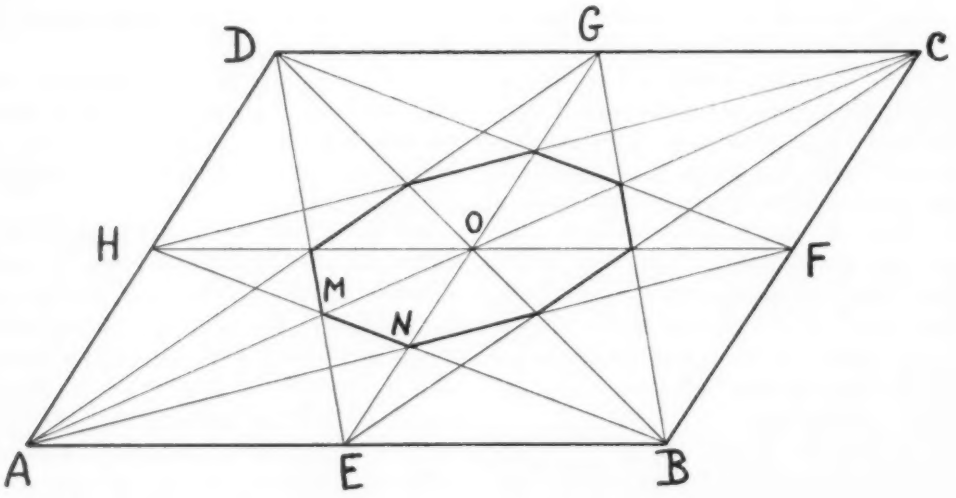
Let us also take into consideration the final result of the original, that is, that the area bounded by the straight lines so drawn is equal to one-sixth of the area of the parallelogram.

Let us restate the above original in a form with is more amenable to the requirements of a puzzling situation. We have a parallelogram. We have the midpoints of the sides. We want to obtain a figure whose area is one-sixth of that of the parallelogram. We will not indicate that the vertices of the parallelogram should be joined with the midpoints of the opposite sides by means of straight lines. But we will impose a definite restriction to the effect that no measurements of any sort are permitted. Thus far we have made only a slight breakaway from the rigid bounds of a geometric original. Now, let us *dress-up* our problem into a more human interest cloak. In other words, let us make up a story and wrap it around the geometric situation. Here is a suggested situation.

Farmer Jones has four sons. He also has a farm whose shape is that of a parallelogram. Farmer Jones decides to divide his farm among his sons so that every one of them will receive an equal acreage. Having reached the ripe age when rest and relaxation are very welcome, Jones plans to subdivide his land before he dies. But knowing how much his supervision of his sons' work is essential in the upkeep of the farm, he feels that he should keep for himself a small portion of the land. He decides to retain one-sixth of the land and that this part of his farm should be centrally located. Now Jones had no use for sur-

veyors. He never had permitted anyone to measure his land. The only markers he ever put on his farm were placed at the corners and at the midpoints of the boundary. How was the division of the land accomplished?

The above restatement of the geometric original may be only one of several other



and probably more effective and intriguing forms of the problem.

As far as the correctness of our assertion that the area of the portion of the parallelogram which is bounded by the straight lines drawn from the vertices of the parallelogram to the midpoints of the opposite sides is equal to one-sixth of the area of the parallelogram this may be established from the following considerations. In triangle ABD the straight lines AO, DE, and BH are medians. Then OM is equal to one third of OA. Now, $ABFH$ is a parallelogram. From this it follows that ON is equal to one half of OE. Compare the area of the triangles AOE and OMN. We have that

$$\text{Area } \triangle OMN = \tfrac{1}{3} \cdot \tfrac{1}{2} \text{ Area } \triangle AOE$$
$$= \tfrac{1}{6} \text{ Area } \triangle AOE.$$

There are eight such triangles in the case of the parallelogram. Thus the area of the inner portion of the parallelogram is equal to one-sixth of the area of the parallelogram.

The readers will note that the inner portion of the parallelogram is an octagon. Furthermore, it is a special kind of an octagon. Its opposite sides are parallel to one another. Dr. Edward Kasner in the book *Mathematics and the Imagination*, pp. 14–16 mentions a polygon on this type and he names it *parhexagon*. There is no reason why we should not call our octagon *paroctagon*. As a matter of fact, any polygon whose opposite sides are parallel to one another is a *parpolygon*. Note that a *parpolygon* must have an even number of sides. Would any one of the readers offer a proof of this statement? A short one, please!

The above problem should offer very few difficulties in a geometry class when the topics on parallelograms and areas are completed. Incidentally, can the readers offer some variations of the above problem in terms of recreational material?

The 30th Annual Meeting of the National Council of Teachers of Mathematics will be held at Des Moines, Iowa, April 16-19, 1952.

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, N. Y.

An Example of the Method of Duplation

NOTE: The following comment is contained in a letter sent by Mr. Hirsch to the editor.

We widh to mention that as far back as the tenth century Rabbi Saadia Gaon, famed philosopher and religious leader[1], made use of the method of duplations (or doubling) to carry out the process of multiplication. One finds in *Scripta Mathematica*, March, 1933,[2] the following description of the method used by the Rabbi to multiply $61+1/3+1/9$ by itself. He proceeds as follows:

$61\times10=610$ $\quad$ $61\times20=1220$

$61\times40=2440$ $\quad$ $61\times60=3360$

$61\times61=3721$

To this he adds the double of 1/3 of 61 and of 1/9 of 61, and approximates the total to 3775.

It will be noted that the multiplication of 61 by itself is performed by the method of duplations. What motivates Rabbi Gaon to add to this product the double of 1/3 of 61 and 1/9 of 61 is not mentioned. The reader will observe that these terms arise in the expansion of $(61+1/3+1/9)^2$ although Newton's statement of the binomial theorem was not yet to be formulated until about 700 years later (1676).[3] If we expand $(61+1/3+1/9)^2$ we have

$$\frac{(61)^2+2\cdot1/3\cdot61+2\cdot1/9\cdot61}{\text{terms in the approximation}}+$$

$$\downarrow$$

$$3775.222$$

$$\frac{(1/3)^2+(1/9)^2+2\cdot1/3\cdot1/9}{\text{terms omitted from the approximation}}$$

$$\downarrow$$

$$+.197=3775.419$$

The Rabbi's approximation of 3775 thus proves to be correct to the nearest integer. It is unlikely that the Rabbi himself was aware of this.

MARTIN HIRSCH
Junior High School 237
Brooklyn, New York

[1] Saadia Gaon (892–942) was head of the academy at Sura in Babylonia at a time when Babylonia in general and the academies at Sura and Pumbedita in particular were the center of Jewish thought. The title "Gaon" meaning excellence was given to the heads of these academies. Rabbi Saadia Gaon, was a scholar of wide achievement and great influence. After his death, the intellectual center of Judaism shifted from Babylonia to Spain. See the *Jewish Encyclopaedia* and also George Sarton's *Introduction to the History of Science*, vol. I,

[2] Harry Polachek, "A Method of Multiplication Used by Saadia Gaon in the 10th Century," *Scripta Mathematica*, Vol. I, (March, 1933) pp. 245–246. The author states that "The mathematical methods used by him (Rabbi Saadia Gaon) are undoubtedly representative of the practices of the leading Jewish and Arabic mathematicians of the time." The computations cited are in connection with the division of inheritances, and beside the problem quoted by Mr. Hirsch, the multiplication of 86 by itself using duplation is given also.

[3] It should be noted here that the binomial theorem was known implicitly for exponents that are positive integers before Isaac Newton made a formula which applied also to exponents that were negative or fractional. In particular, the square of a binomial is stated in geometric form by Euclid in Proposition 4, Book II in these terms:

"If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments."

This in itself does not establish that Rabbi Saadia was acquainted with Euclid's work, but the fact that the Elements was one of the first books translated from the Greek into Arabic and that the translation was made at the request of the Caliph of Bagdad, would make it seem more than possible that a Hebrew scholar living in Babylonia would have been aware of this classic. That, however, is pure conjecture.

BOOK SECTION

Edited by JOSEPH J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

ELEMENTARY SCHOOL

Numbers in Action, Teachers Edition, by Maurice L. Hartung, Henry Van Engen, and Catharine Mahoney. Cloth, 144 pages+Instructions to the Teacher, 1951. Scott, Foresman and Company, 433 E. Erie Street, Chicago 11, Illinois.

COLLEGE

1. 1st Year Mathematics

College Mathematics, by Charles E. Clark, Emory University. Cloth, v+331 pages, + tables, 1950. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N.Y. $3.85.

2. Trigonometry

Plane Trigonometry, with Tables, by Lyman M. Kells, Willis F. Kern, and James R. Bland, all at the United States Naval Academy. Cloth, xi+220 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N.Y. $3.50.

3. Statistics

Statistical Methodology Reviews, 1941–1950, Edited by Oscar Krisen Buros, Rutgers University. Cloth, x+457 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N.Y. $7.00.

4. Special Courses in Mathematics

Mathematics for Engineers, 3rd Edition, by Raymond W. Dull and Richard Dull. Cloth, xix+822 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N.Y. $7.50.

Mathematics of Finance, by Albert E. May, University of Wisconsin. Cloth, viii+264 pages, 1951. American Book Company, 55 Fifth Avenue, New York 3, N.Y. $3.00.

5. Advanced Mathematics

The Albegra of Vectors and Matrices, by Thomas L. Wade, Florida State University. Cloth, ix+189 pages, 1951. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $4.50.

TEACHING OF MATHEMATICS

Developing Meaningful Practices in Arithmetic, A Third Report by the Committee on Flexibility, Central New York School Study Council. Paper, xii+123 pages, 1951. Central New York School Study Council, 219 Slocum Hall, College Place, Syracuse 10, N.Y. $2.00.

Vitalizing Secondary Education, The First Commission on Life Adjustment Education for Youth. Paper, 106 pages, 1951. Federal Security Agency, Office of Education, Bulletin No. 3. Available through the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.30.

MISCELLANEOUS

Anthology in Educology, by Lowry W. Harding, Ohio State University. Paper, xv+78 pages, 1951. Wm. C. Brown Company, Dubuque, Iowa. $1.50.

How to Study—How to Solve, by H. M. Dadourian, Trinity College. Paper, vi+121 pages, 1951. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $.60.

REVIEWS

Arithmetic 8: The World of Numbers, Dale Carpenter and George F. Drake, Jr. New York, The Macmillan Company, 1950. 372 pp., $1.68.

The outstanding physical characteristic of this book is the excellent use of bold type—superior in size and spacing.

The simplicity of the wording of the table of contents and index make them satisfactory guides for both student and teacher.

The topics developed in this text are "Whole Numbers and Common Fractions," "Decimal Fractions," "Percentage," "Measurements," "Using Geometry," "Mensuration," "Using Your Income Wisely," "Arithmetic in the Home," and "Using Algebra."

Sections on "Factors" and "Rounding Off" are especially clearly treated by the authors. The stress on vocabulary is a noteworthy feature of this book. In addition to this it contains other desirable aids to learning such as practice exercises, remedial tests, and tables of measures. Magic squares are included as enrichment material.

While this book gives an understanding of how number ideas work and stresses meaning, it does not, however, provide ample opportunity for developing speed and skill through many practice exercises.

The authors could have dispensed with Chapter IX, "Using Algebra," and could have

utilized this space more advantageously for more work in arithmetic.

I would recommend the use of this book in a slow eighth grade class.—GERTRUDE HOLZAPFEL, Hamilton Junior High School, Baltimore, Maryland.

First Algebra, Virgil S. Mallory. Chicago, Benj. H. Sanborn and Co., 1950. vii+444 pp., $2.24.

This textbook for ninth year algebra classes is not a revision of Mr. Mallory's earlier texts but, in his words, is "the culmination of many years of experience in teaching high school students, of consultations with teachers throughout the United States and in the observation of the teaching of algebra in many high school classes." It is written in clear concise language with new topics developed with the concepts which the pupil understands.

Each type of difficulty is given in boxed examples with full explanations followed by "Test Yourself" exercises with the answers given. In fact, the text seems to be as nearly self-teaching as it is possible to achieve.

A few pages of story problems are given in practically every chapter. These are about everyday life situations that are familiar to the ninth year pupil which leads to easy understanding of equations and formulas.

Most of the exercises are divided into three levels of difficulty: one covers the minimum requirements; the second demands somewhat greater skill; the starred problems challenge the student who is capable of doing superior work. Exercises of various types which are amply supplied make work books and supplementary problems unnecessary. The eight pages of remedial practice in arithmetic at the end of the book as well as diagnostic tests throughout the book encourages continued proficiency in arithmetic.

Cumulative and chapter reviews, keyed practice material and chapter tests furnish invaluable aids for the busy teacher. Numerous historical notes, accounts of modern mathematical applications and illustrations of topics of interest are included.

The combination of good format and large type make easy ninth grade reading.—ELIZABETH VAN LIEW, Mooresville High School, Mooresville, Indiana.

Plane Trigonometry, Alternate Edition. E. Richard Heineman. New York, McGraw-Hill, 1950. xiv+184 pp. and tables. $2.50.

Professor Heineman has written another fine textbook on trigonometry—especially for those students whose mathematical background is meager. The principal objective of the author has been teachability. The reviewer finds that the textbook bears testimony to the author's conscientious observance of this goal.

The contents are improved by the new rule for finding the characteristic of a logarithm, and a worthwhile note to the students. The subject matter of trigonometry is well covered and topics that are omitted in the textbook are those which are usually omitted in a high school course, anyway, due to lack of time.

Answers are given to all problems except those whose numbers are multiples of four. For the convenience of teachers, brief outlines for two-semester-hour and three-semester-hour courses are given.—ELBERT C. HUBBARD, Parks College, East St. Louis, Illinois.

College Algebra, Herman K. Fulmer and Walter Reynolds. Boston, Ginn and Company, 1951. vi+218 pp., $2.85.

The authors of this text have written a really brief college algebra. They have condensed into less than two hundred pages all the topics usually found in books twice the length except permutations, combinations and probability which have been omitted.

No attempt is made to prove or even to rationalize any of the operations discussed in the first six chapters. In the remaining chapters most proofs are indicated but the authors make liberal use of the rules illustrated by examples.

The chapter on Theory of Equations contains elementary topics through Horner's Method and Cramer's Rule is applied to systems of three equations in three unknowns only.

There are nearly fourteen hundred exercises seemingly well selected and well graded. The answers to all of the exercises are given in the back of the book. A four place logarithm table is included. The print is large and clear and the general appearance is attractive.—FRANK C. GENTRY, Arizona State College, Tempe, Arizona.

Lecture on Classical Differential Geometry, D. J. Struik. Cambridge, Addison-Wesley Press, Inc., 1950. viii+221 pp., $6.00.

This is an excellent account of the classical theory of differential geometry. The book is an outgrowth of a course which the author teaches at the Massachusetts Institute of Technology and is designed as a textbook, with an ample supply of problems. There is also a large collection of diagrams and photographs of models.

Anyone who is thoroughly grounded in advance calculus has adequate preparation to read the book; a knowledge of the rudiments of vector analysis would be highly desirable, but it can probably be picked up from the text.

The subject of the book is the application of calculus to the study of geometry of curves and surfaces. The presentation differs from that of many textbooks in English in that vector methods and vector notation are used throughout; the reader who has studied differential geometry without the use of vectors will be startled by the simplification that results.

The author has included numerous thumbnail biographies of the men who have contributed to the development of the subject.—M. P. GAFFNEY, Northwestern University, Evanston, Illinois.

The Arithmetic Theory of Quadratic Forms (Carus Monograph Number 10), Burton W. Jones. Buffalo, The Mathematical Association of America, 1950. x+212 pp., $3.00.

The author presupposes knowledge of the fundamentals of matrix theory and elementary number theory. Otherwise, the book gives a self-contained presentation of the central ideas of the arithmetic theory of quadratic forms. It begins with the classical theory in the field of reals from the point of view of representation theory, and successively narrows the fields and rings of coefficients until the classical problems in the ring of rational integers can be dealt with. Results for binary and ternary forms are then obtained as special cases of the preceding general theory. Use of the concepts of p-adic numbers and ideals in quadratic fields adds elegance to the presentation.

A comprehensive set of computational and theoretical problems is included after the bibliography. The general purpose of these problems seems to be clarification rather than extension of the theory.—T. C. Holyoke, Northwestern University, Evanston, Illinois.

Coefficient Regions for Schlicht Functions, A. C. Schaeffer and D. C. Spencer. New York, American Mathematical Society, 1950. xv+311 pp., $6.00.

This monograph is Volume 35 in the Colloquium Publications series and is a presentation of the results which the authors have obtained in the investigation of the family of extremal schlicht functions in the large. Most of the results are new. An additional chapter entitled "The region of values of the derivative of a schlicht function" by Arthur Grad has been included in the book.

A function $f(z)$ is said to be schlicht in a domain if, for any two points z_1 and z_2, $f(z_1) = f(z_2)$ only if $z_1 = z_2$. The investigation is concerned with the class of functions $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ which are regular and schlicht in the unit circle $|z| < 1$. More specifically, the problem is to determine the possible values of $a_2, a_3, \cdots$.

Necessary background for reading this monograph is a knowledge of the material contained in a standard course on functions of a complex variable.—D. H. Potts, Northwestern University, Evanston, Illinois.

Methods and Materials for Teaching General and Physical Science, John S. Richardson and G. P. Cahoon. New York, McGraw-Hill Book Company, 1951. viii+485 pp., $4.50.

This book is a companion volume to Miller and Blayd's in the biological sciences. The treatment is concerned almost entirely with student activities as related to apparatus and materials in physics, chemistry, and general science. For science teachers, the detailed description of projects, of classroom demonstrations, and of individual laboratory experiments, make the book a veritable mine of source materials. The careful explanations of classroom methods will enable the teacher to attain, to a much greater extent than otherwise, the author's objectives of this type of teaching, namely an experiential, meaningful learning, motivated and adapted to the various individual abilities, to create critical thinking, as well as initiative, resourcefulness, and cooperation. For the beginning teacher the book is an almost indispensable guide; for teachers of mathematics who are interested in laboratory procedures, the book suggests and opens many avenues of approach. The chapter on teaching for thinking through laboratory experiences will be of special significance to mathematics teachers. The lack of applied mathematics (there is very little of it) in explaining most of the physical properties, and the results of laboratory experiments, will appear as a defect to most teachers of mathematics. This may be explained in part by the author's major concern for general education rather than for college preparatory education.

Part I of the text treats: planning the use of apparatus, using demonstrations, using student projects and field trips, providing and using visual materials, and providing and care of equipment. Part II gives specific demonstrations, laboratory procedures, and projects for general science, physics and chemistry classes.

The format of the book is excellent. The print, the drawings, the paper, all make the book easy to use. There is a brief but pertinent bibliography, and an excellent index.—Howard F. Fehr, Columbia University, New York.

Differential Algebra, Joseph Fels Ritt. New York, American Mathematical Society, 1950. v+184 pp., $4.40.

Professor Ritt's book is volume 33 in the Colloquium Publications series of the American Mathematical Society. The subject considered is the study of the algebraic structure of the theory of certain types of differential equations, namely those which are polynomials in the unknown function and its derivatives, the coefficients being from a certain type of algebraic field. The present work goes back to the author's earlier *Differential Equations from the Algebraic Standpoint* (Colloquium Publications, Vol. 14, 1932). In the ensuing years the author and his associates have advanced the subject to such a degree that the present work must be regarded as more than a revision and enlargement of the earlier publication.

The problems considered are those of the classical theory of differential equations, but the abstract method employed serves to separate the algebraic from the analytic and, as abstract methods usually do, serves to give the subject a coherence and unity which the classical methods cannot produce.

As to necessary background required, the exposition should be within the reach of a ma-

(*Continued on page 598*)

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Book Section

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ture person skilled in the reading of mathematical notation provided that he has some knowledge of classical algebra, differential equations, and a smattering of abstract algebra.—D. H. Potts, Northwestern University, Evanston, Illinois.

The Theory of Algebraic Numbers, Harry Pollard. New York, John Wiley and Sons, Inc., 1950. xii+143 pp., $3.00.

This volume is number nine in the series of Carus Mathematical Monographs. These monographs are of expository nature and are designed to be comprehensible to "thoughtful people having a moderate acquaintance with elementary mathematics." This particular monograph fits the bill rather well, as much of it could be digested by a person who has had a second year of algebra, although there could be some notational difficulties.

The subject of the book is indicated by the title. More specifically, the material presented constitutes the elementary parts of classical algebraic number theory. Briefly an algebraic number is a root of a polynomial equation with rational coefficients. (The concept can be extended to equations with a more general type of coefficients.) It is proved that there exist nonalgebraic numbers (i.e. transcendental numbers). The book not only introduces a subject which is fascinating in itself but also would serve as a fine background for courses in modern algebra.—D. H. Potts, Northwestern University, Evanston, Illinois.

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